

Calculating Colour Distance on Choropleth Maps with Sequential Colours – A Case Study with ColorBrewer 2.0

Die Berechnung von Farbdistanz in Choroplethenkarten mit sequenziellen Farben – Eine Fallstudie mit ColorBrewer 2.0

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In this paper, we first present a procedure derived from related vision and perception literature to calculate the colour metric ΔE_{00} , as our previous empirical research let us believe that this is a reliable metric that can be useful in cartographic design decisions (Brychtová and Çöltekin, 2015; 2016). In earlier work, we demonstrated that increasing ΔE_{00} values consistently improves the human judgement of whether two colours are the same or different both with sequential and qualitative schemes. Furthermore, we observed that colour distance $\Delta E_{00}=10$ 'works' in terms of same/different judgements for two colours, even if the compared colours are (spatially) far apart. Using this knowledge from previous work of others as well as our previous own work, we evaluate a subset of colours used in the well-known online colour recommender ColorBrewer 2.0 against $\Delta E_{00} = 10$ threshold as a minimum perceptually safe colour distance. The results of the evaluation showed that overall, majority of the evaluated colours are equal to or larger than the perceptually safe $\Delta E_{00} = 10$, however, there are also colour distances that are considerably lower. These findings suggest that some widely adopted colour schemes might not be ideal under some circumstances, and call for more research.

■ Keywords: Colour, Colour Distance, Colour Scheme, ColorBrewer 2.0, Choropleth Map

In diesem Beitrag wird eine Vorgehensweise zur Berechnung der Farbmessung ΔE_{00} vorgestellt, die aus benachbarter Fachliteratur der Blick- und Wahrnehmungsforschung abgeleitet ist. Bereits vorausgehende empirische Studien deuten darauf hin, dass hinter dieser Metrik ein zuverlässiges Verfahren steht, das kartographische Gestaltungsüberlegungen unterstützen kann (Brychtová und Çöltekin 2015; 2016). In diesen vorausgehenden Studien wurde gezeigt, dass die Erhöhung von ΔE_{00} -Werten das Unterscheiden von zwei Farben in sequenziellen und qualitativen Farbschemen verbessert. Dabei wurde auch beobachtet, dass Farbdistanz ($\Delta E_{00} = 10$) hinsichtlich gleicher und unterschiedlicher Bewertung zweier Farben funktioniert, auch wenn die verglichenen Farben (räumlich) weit auseinanderliegen. Auf der Grundlage dieser Erkenntnisse wird in dieser Studie eine Auswahl an Farben untersucht, die den weit verbreiteten Farbpfehlungen des Online-Tools ColorBrewer 2.0 entnommen sind. Diese Untersuchung berücksichtigt den Schwellwert $\Delta E_{00} = 10$ als minimal sicher wahrnehmbare Farbdistanz. Die Resultate zeigen, dass die meisten untersuchten Farben dem Farbdistanzschwellwert entsprechen oder über ihm liegen. Es gibt jedoch auch Farbdistanzen, die beachtlich unter dem Schwellwert liegen. Daraus ist abzuleiten, dass manche weit verbreiteten Farbschemen zu optimieren sind, was weitere empirische Studien erfordert.

■ Schlüsselwörter: Farbe, Farbdistanz, Farbschema, ColorBrewer 2.0, Choroplethenkarte

1 Introduction

In cartography, colour is used for depicting important information and it is a key visual variable for communicating the quality and quantity of visualized spatial information (Bertin 1983). Being able to match or discriminate colours is very important for visually identifying information, e. g. patterns or anomalies. This visual identification of patterns and anomalies depend on whether we can tell if the two (or more) shades are the same or different; and being able to

identify such information is, at a very fundamental level, a perceptual process. *Color distance* – a metric intended to quantify differences in a viewer's visual perception of two colour stimuli – has considerable impact on the overall map readability. Recent empirical evidence confirms that increasing colour distance has a consistent positive impact on the ability to differentiate colours within both sequential and qualitative schemes (Brychtová and Çöltekin 2015; 2016). These studies were conducted using the metric ΔE_{00} (elaborated in detail in later

sections), and results clearly demonstrated that $\Delta E_{00} = 10$ yields considerably higher levels of accuracy for discriminating colours than other studied colour distances with lower ΔE_{00} values (Brychtová and Çöltekin 2015; 2016).

In this article, we first provide the procedure (including the mathematical model) we followed to obtain our ΔE_{00} values. This procedure is synthesized from the fundamental knowledge found in literature; however, providing it in a concise manner in this paper could benefit those who wish to study

colour distance themselves (possibly most relevant to those who are new to the subject). Furthermore, given that many cartographers rely on software recommendations to select their colour schemes, and do not manually check for ΔE_{00} values for each shade, we examine a subset of the colour distances found on the most popular colour recommender, *ColorBrewer 2.0* (<http://colorbrewer2.org>). *ColorBrewer 2.0* is used by a large number of people including researchers and practitioners in interdisciplinary visualization communities (cartography, information visualization, scientific visualization), providing a great variety of colour schemes. Our goal is not to evaluate *ColorBrewer 2.0* per se, but because this tool is very popular, we believe examining a sample of colour schemes found in *ColorBrewer 2.0* might reflect (approximately) the state of the art in cartography practice from the lens of ΔE_{00} as a colour distance metric. Thus, after providing our procedure to obtain ΔE_{00} , we continue on to an analysis in which we ‘reverse engineer’ a subset of the colour schemes used in *ColorBrewer 2.0* and obtain the ΔE_{00} values for each. Furthermore, contrasting them with our experimental results allows us to reflect on our recommended ΔE_{00} thresholds, and discuss them within a wider context than previously user-tested colour schemes in this context.

ColorBrewer 2.0 offers 18 sequential, 9 divergent and 8 qualitative colour schemes of 3 to 12 classes. The shades of colour schemes were selected from Munsell colour charts (Brewer 1989), thus their differences roughly correspond to the human colour perception. Munsell colour charts represent one of the earliest (possibly the first) successful definitions of a perceptually uniform colour space. However, since the proposition of Munsell charts, there have been many developments. Munsell colour order system was designed by Albert Munsell, an American painter and art teacher (Landa and Fairchild 2005). Munsell colour patches are described with *hue*, *chroma*, and *value*. Based on his own measurements, Munsell has shown that the sensitivity of the human eye is not homogenous across the colour spectrum. For various levels of hue and value, there are different amounts of chroma levels. This is reflected in the irregular shape of the model on which one can see that e. g. the lightest possible shade of green appears much lighter than the lightest possible shade of red (X-Rite 2012). After some adjustments, Munsell colour system is still used in the practice today.

2 Related work

2.1 Colour perception

Research on colour perception appears to have been active since the late 18th century (Google Ngram 2017). Describing the precise mechanisms of human colour perception is attributed to Thomas Young at the beginning of the 19th century (Gegenfurtner and Sharpe 2001). Our modern understanding is that the colour vision is created by the reactions of three types of light-sensitive cells (photoreceptors, particularly cones) on the retina to the incident light (Gegenfurtner and Sharpe 2001). However, colour perception is far from fully understood, and remains ‘tricky’ as individual or circumstantial differences can affect the colour perception (Lafer-Sousa et al. 2015; Xiao et al. 2016). Human colour perception is highly dependent on environmental as well as psycho-physical (biological and cognitive) factors (May 2009). There are strong differences in the way humans experience colour (Asano, Fairchild, Blondé, & Morvan 2015). For example, the perception of colour is strongly affected by various factors such as the amount of the light in the environment, objects casting shadows, surrounding materials and their reflectivity as well as observers’ previous knowledge and cognitive biases (Derefeldt, Swartling, Berggrund, & Bodrogi 2004; Foster 2011). Furthermore, it is well-documented that the number and distribution of photoreceptors in the eye influences what we see (Roy et al., 1991) as well as (arguably) our brain assuming certain light direction or source (e. g. Gegenfurtner et al. 2015; Lafer-Sousa et al. 2015; Winkler et al. 2015). In summary, we understand that colour perception is not *stable* over space and time for one individual; nor is it between individuals or groups. Despite this instability, there are a number of efforts to model and quantify colour perception. These efforts include mathematical models that attempt establishing the thresholds at which we can tell two colours (or the shades of the same colour) apart. Being able to tell apart colours and shades of the same colour is of central interest in cartography (and is the core topic for this paper). *Colour distance* is such a metric that quantifies human abilities to visually distinguish differences between two colours. This metric was introduced by the *International Commission for Illumination* (CIE, in French *Commission Internationale de l’Éclairage*). Colour distance is denoted as ΔE , where *delta* (Δ) refers

to the difference and *E* stands for the German term *Empfindung*, translated as *Sensation* (Robertson 1990).

2.2 Colour research in cartography

Even though colour has been a central topic for cartographic research from a design perspective for a long time, there seems to be little research on empirical determination of the minimum effective colour distance to distinguish cartographic symbols. The most considerable contribution to colour research in modern cartography is by Brewer and her colleagues (Brewer et al. 2003; Brewer 1986, 1992, 1994, 1996, 1997, 1999; Harrower and Brewer 2003). Brewer and her colleagues developed a set of colour schemes for qualitative and quantitative data visualization, and a very helpful online software which is well known – in and beyond cartography – as *ColorBrewer 2.0* (<http://colorbrewer2.org/>). Brewer and her collaborators designed colour schemes to maintain consistency in the perceived colour distances between classes using Munsell charts. Our previous own work, based on various controlled and online experiments, clearly demonstrates that even subtle manipulation of colour distance has a considerable impact on the overall map readability (Brychtová and Çöltekin 2016; Brychtová and Çöltekin 2015; Brychtová and Çöltekin 2014; Brychtová 2014; Brychtová and Vondráková 2014). Generally, we see that increasing the spatial distance between two mapped areas of certain colours has a consistent negative impact on the ability to differentiate them within both sequential and qualitative schemes (Brychtová and Çöltekin 2016). Furthermore, in the same study, we demonstrated that colour distance $\Delta E_{00} = 10$ yields considerably higher levels of accuracy in the colour discrimination, even if the spatial gap between the two colours is relatively large. Therefore, this “more conservative” colour distance can be recommended when designing sequential schemes. However, the best results were observed for colour schemes of six classes with a concave distribution of colour distances (namely $\Delta E_{00} = 4-8-10-8-4$) (Brychtová and Çöltekin 2016; Brychtová and Çöltekin 2015). An important question here is why ΔE_{00} should be considered a ‘good’ measure. In the next section, we summarize some of the fundamental knowledge about colour model spaces and mathematical equations needed to calculate the colour difference as expressed by ΔE_{00} .

2.3 Colour distance: Calculation of ΔE_{00}

The approach to mathematically describe a colour stimulus is referred to as *colour modeling system* (Levkowitz 1997; Robinson et al. 1995), or simply, *colour model*. There is plethora of colour models representing the logic of creating a colour (Kuehni, 2001). They can be divided into 4 main groups (adjusted according to (Levkowitz 1997)): *instrumental* (e. g. RGB or CMYK), *pseudo-perceptual* (e. g. HLS, HSV or HSB), *colourimetric* (e.g. 1931 CIE XYZ) and *perceptually uniform* (e.g. Munsell system, CIELAB or CIELUV) colour models. The set of all existing colours which can be reproduced by a concrete system (e. g. screen, printer or a human eye) originating from the combinations of a colour model components is referred to as the *colour space* (also known as *colour gamut*). Colour space is defined based on a *reference colour space*, which represents a standardized description of the human perception of colour under certain lighting conditions. The most widely used reference colour spaces are CIE 1931 XYZ or CIELAB. Lighting conditions are defined through the temperature of a reference white light – e. g. daylight D65 of colour temperature 6500 K (Pascale 2003). Examples of colour spaces based on RGB colour model are sRGB, Adobe RGB or ProPhoto RGB, whose three model components (R, G and B) are defined by the CIE 1931 XYZ colour space coordinates, and by the reference white light D65. Colour stimulus defined only with R, G and B values without reference values could take virtually any form. To calculate colour difference corresponding to the human perception, referencing to colour spaces derived from perceptually uniform spaces fundamental. In the perceptually uniform space, certain distance corresponds to the perceived distance of the same size (CIE, 2012). In other words, certain change of the colour in the perceptually uniform space produces equal change in human perception of that colour (Slocum et al. 2008).

3 Procedure for calculating the ΔE_{00}

In this section we present a complete step-by-step procedure to calculate the colour distance between two colour shades based on a perceptually uniform colour space, which we synthesized from the literature. This approach is what we implemented (as explained in Brychtová and Doležalová 2015) and tested in various user experiments (Brychtová and Çöltekin 2016, 2015, 2014;

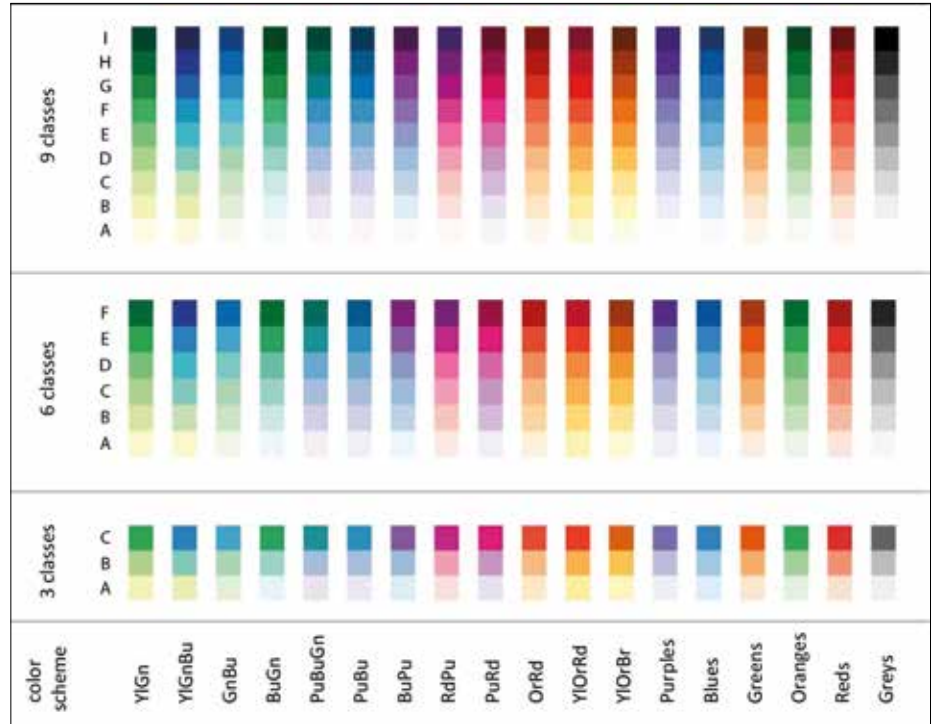


Fig. 1: Overview of 18 sequential colour schemes variants of ColorBrewer 2.0 with 3, 6 and 9 classes. Adapted from ColorBrewer 2.0. Individual shades are depicted with letters A to I from lightest to the darkest shade

Brychtová 2014, Brychtová and Vondráková 2014).

3.1 Step1: Relating the relative colour model values to absolute colour space components

Colours for digital maps are most often defined through components of the RGB colour model, as it represents the logic how colours are created on the computer screen. Its principle is based on composing three colour components of varying intensity: red (R), green (G) and blue (B) in an additive manner to create the other colours. As we

already noted, however, RGB values alone do not refer to any colour if not related to an absolute colour space. Thus, in the first step, we have to reference the relative RGB values to an absolute colour space. For the absolute colour space, we choose the sRGB, which was developed in 1996 for viewing graphics on the Internet (Stokes et al. 1996), and is considered to be the standard space, as its gamut can be displayed by most commonly used screens. The sRGB is described by standard IEC 61966-2-1:1999, and is adopted by modern browsers: Without relating RGB values to any other absolute colour space, RGB colours are displayed in

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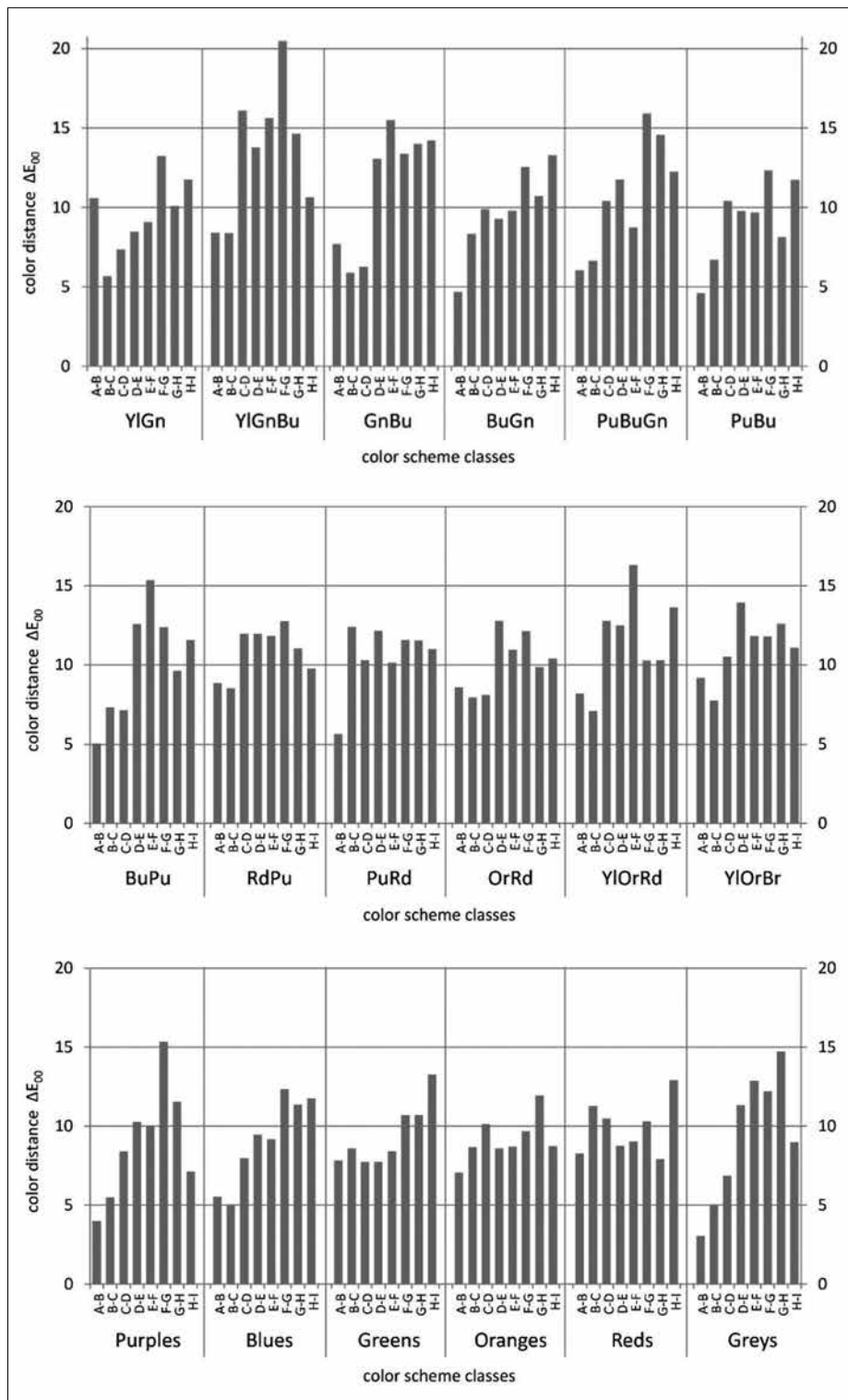


Fig. 2: Colour distances in ColorBrewer 2.0 colour schemes of 9 classes (A–I)

web browsers if they are sRGB. For our ‘recipe’ to calculate the ΔE_{00} , it is not critical that sRGB is used, that is, we could reference the RGB codes to whatever arbitrary absolute colour space (such as AdobeRGB, ProPhotoRGB, colour space of my or your screen). We choose the sRGB because the chance that our results are valid for most screens and web browsers is the highest (not many screens have the capacity to show the

whole gamut of AdobeRGB or ProPhotoRGB). The process is straightforward, as it does not require any mathematical transformation. We basically took the given RGB and gave them the absolute meaning by deciding that given values are representing the sRGB (e. g. RGB model [0.15, 0.91, 0.87] = sRGB [0.15, 0.91, 0.87] = ProPhotoRGB [0.15, 0.91, 0.87] = WhateverRGB [0.15, 0.91, 0.87]).

3.2 Step2: Transforming sRGB to CIE XYZ

In the next step, we transform the absolute colour space coordinates to the reference colour space CIE 1931 XYZ. The CIE XYZ 1931 model defines the qualitative relationship between the spectral colours (wavelengths) in the electromagnetic visible spectrum and physiologically perceived colours by the average human eye (Fairchild 2013). The model was derived from a series of experimental measurements. Specifically, participants were instructed to report a match (i. e. metamerism) between monochromatic colours (defined in the wavelength range of 380 nm to 780 nm) and the colour mixture of three monochromatic components (red $\lambda_r = 700$ nm, green $\lambda_g = 546.1$ nm and blue $\lambda_b = 435.8$ nm), while they adjusted their light intensity (Fairchild 2013). The transformation of sRGB [R,G,B] to CIE 1931 XYZ [X,Y,Z] can be realized via set of equations [1] (Lindbloom, 2012):

$$R_{lin} = \begin{cases} \frac{R}{12.92} & \text{if } R \leq 0.04045 \\ \sqrt[5]{\frac{R+0.055}{1+0.055}}^{12} & \text{otherwise } R > 0.04045 \end{cases} \quad [1]$$

$$G_{lin} = \begin{cases} \frac{G}{12.92} & \text{if } G \leq 0.04045 \\ \sqrt[5]{\frac{G+0.055}{1+0.055}}^{12} & \text{otherwise } G > 0.04045 \end{cases}$$

$$B_{lin} = \begin{cases} \frac{B}{12.92} & \text{if } B \leq 0.04045 \\ \sqrt[5]{\frac{B+0.055}{1+0.055}}^{12} & \text{otherwise } B > 0.04045 \end{cases}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{bmatrix} \begin{bmatrix} R_{lin} \\ G_{lin} \\ B_{lin} \end{bmatrix}$$

where R_{lin} , G_{lin} , B_{lin} are values after reverse gamma correction and they assume values from the interval (0;1), input coordinates sRGB [R,G,B] assume values from the interval (0;1) as well. Given that relation [1] includes recalculations towards standard reference white point D65 and gamma correction delinearizing colour value with $\gamma = 2.2$.

3.3 Step 3: Transforming CIE XYZ to CIE Lab

The colour model CIE 1931 XYZ itself does not meet the conditions of perceptual uniformity (“Certain change of the colour in the perceptually uniform space produces equally perceptive change (Slocum et al., 2008)”). Thus, to be able to calculate the colour distance between the two shades we have to perform another transformation. We chose to work with the CIELAB colour space (also

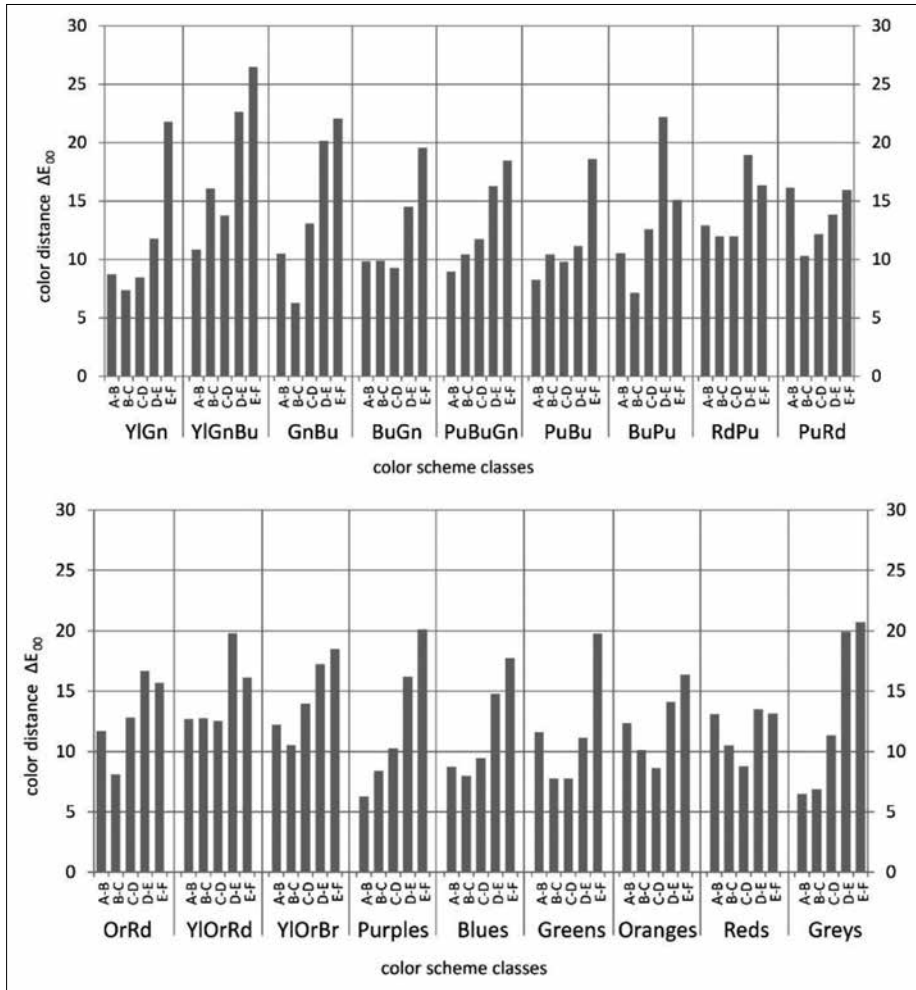


Fig. 3: Colour distances in ColorBrewer 2.0 colour schemes of 6 classes (A–F)

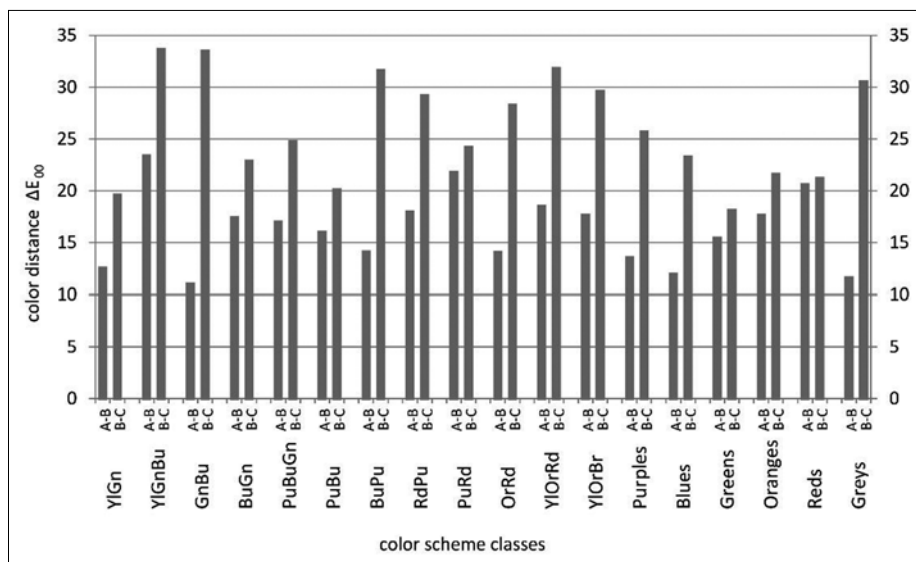


Fig. 4: Colour distances in ColorBrewer 2.0 colour schemes of 3 classes (A–C)

known as CIE 1976 (L^* , a^* , b^*) in this step. The CIELAB colour model describes all colours perceptible by humans, and is therefore device independent (Levkowitz, 1997). CIELAB coordinates are nonlinear functions of the CIE 1931 XYZ, and are dependent on the specifications of the white point (CIE 2014b).

The CIELAB model consists of three components: L represents the brightness of colour ($L = 0$ indicates black; $L = 100$ indicates a diffuse white); a represents the axis between red and green (negative values indicate green while positive values indicate magenta), b represents the axis of the blue and

yellow (negative values indicate blue and positive values indicate yellow). Components a and b can theoretically have any real value, but practically they are limited by the human factor. The relationship between the CIE 1931 XYZ [X, Y, Z] and CIELAB [L, a, b] colour models can be expressed via a set of equations [2]:

$$L = 116f\left(\frac{Y}{Y_n}\right) - 16$$

$$a = 500\left[f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right)\right]$$

$$b = 200\left[f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right)\right]$$

where

$$f(t) = \begin{cases} \sqrt[3]{t}, & \text{if } t > \left(\frac{6}{29}\right)^3 \\ \frac{1}{3}\left(\frac{29}{6}\right)^3 t + \frac{4}{29}, & \text{otherwise} \end{cases}$$

and X_n, Y_n, Z_n are coordinates of the reference white point.

3.4 Step4: Calculating the colour distance ΔE_{00}

Finally, in the last step, we calculate the colour distance ΔE_{00} between the two shades of the same colour. Currently, CIEDE2000 (ΔE_{00}) is one of the most precise methods to calculate colour distance (Werman 2012; Carter and Huertas 2009; Yang et al. 2012). CIEDE2000 is based on CIELAB colour space, however contains compensation for neutral colours, lightness, chroma and hue to reach higher perceptual uniformity. This method was previously shown to be suitable for calculations of both small ($\Delta E_{00} < 1$) and large ($\Delta E_{00} > 10$) colour distances (Carter and Huertas 2009). The method is described by equation [3] (Sharma et al. 2005).

$$\Delta E_{00} = \sqrt{\left(\frac{\Delta L'}{k_L S_L}\right)^2 + \left(\frac{\Delta C'}{k_C S_C}\right)^2 + \left(\frac{\Delta H'}{k_H S_H}\right)^2 + R_T \frac{\Delta C'}{k_C S_C} \frac{\Delta H'}{k_H S_H}}$$

where k_L, k_C, k_H are parametric coefficients adjusting the equation according to observer environment; S_L, S_C, S_H are weighting coefficients for brightness, saturation and hue respectively, and R_T rotary factor adjusting the variation in the blue region (Luo et al. 2001). Then $\Delta L', \Delta C', \Delta H'$ and R_T hold as follows [4].

Based on the procedure and formulae above, thus, ΔE_{00} is calculated. A reverse approach to design colour schemes based on the given colour distance between its classes is also

[4] possible. A software implementation of this approach can be found at <http://eyetracking.upol.cz/color/> (as described in Brychtová and Doležalová 2015).

4 Case study: Evaluation of colour distances in ColorBrewer 2.0

As we reported earlier in this article, based on previous empirical evidence, we are convinced that a value around $\Delta E_{00} = 10$ is safer than smaller colour distances for correctly identifying if two colours are same or different; especially when the colours are separated by a spatial distance larger than 6° of visual angle (Brychtová and Çöltekin 2016). While more research would further solidify these results (for example, with wider thresholds and finer variation in the ΔE_{00} values, with different visualization parameters such as finer spatial distances, more colour schemes, or in the presence of various colour interaction effects such as colour constancy, simultaneous contrast or chromatic adaptation), we believe $\Delta E_{00} = 10$ is recommendable as a basic 'safe' colour distance to ensure perceptual robustness in a choropleth map. However, it is not clear whether this threshold is already in use, and if not, what is being recommended by other systems at the moment as measured by ΔE_{00} . Thus, to understand if this threshold is used in practice, we examine a subset of colour schemes offered by *ColorBrewer 2.0*, arguably the most popular colour software today. Specifically, we selected 18 sequential schemes of 3, 6 and 9 classes (Figure 1). This selection was done for practical reason to limit the amount of published results: 3 classes are the minimum what *ColorBrewer 2.0* provides for sequential schemes, while 9 classes is the maximum and 6 is the mean. With this sample, we believe we

can investigate the largest, mean and smallest colour distances.

ColorBrewer 2.0 provides a specification of colours through HEX, RGB or CMYK values. We calculated the colour distance between adjacent colour shades following the 4-step procedure described in the previous section. As Figures 2, 3 and 4 show, the colour distance between adjacent classes is not constant within individual schemes.

The distribution of colour distances also varies across all colour variants. Mostly, the colour distance increases toward the darker shades (ΔE_{00} is larger between darker shades than between lighter shades). This is particularly apparent for the colour schemes with 3 classes (all colour variants) and 6 classes (e. g. YlGnBu, PuBuGn, Purples, Blues or Greys). Opposite distribution (decreasing colour distance towards darker shades) is absent. Colour schemes with 9 classes show higher colour differences between middle classes (e. g. YlGnBu, BuPu, RdPu or Purples). The opposite distribution (smaller colour distance between middle classes) is on colour schemes PuRd, Oranges and Greens with 6 classes. Shades of colour schemes with larger number of classes are generally less distinct than the schemes with fewer classes.

The median colour distance ΔE_{00} in colour schemes of 9 classes is $\Delta E_{00}Mdn = 10.28$ ($\Delta E_{00}min = 3.04$; $\Delta E_{00}max = 20.46$; Figure 2), of 6 classes is $\Delta E_{00}Mdn = 12.41$ ($\Delta E_{00}min = 6.24$; $\Delta E_{00}max = 26.44$; Figure 3), while of 3 classes it is $\Delta E_{00}Mdn = 20.61$ ($\Delta E_{00}min = 11.26$; $\Delta E_{00}max = 33.92$; Figure 4). The most frequent colour distances are for schemes of 9 classes in the range $\Delta E_{00}(8;11)$, of 6 classes $\Delta E_{00}(10;11)$ and for 3 classes $\Delta E_{00}(17;18)$; see Figures 5, 6 and 7.

Overall, we see that most frequent colour distances in the studied schemes of *ColorBrewer 2.0* are in the range $\Delta E_{00}(8;11)$ for 9 classes, $\Delta E_{00}(10;11)$ for 6 classes, and $\Delta E_{00}(17;18)$ for 3 classes.

5 Discussion and conclusions

In this paper, we first presented a procedure derived from related vision and perception literature to calculate the colour metric ΔE_{00} , as our research let us believe that this is a reliable metric that can be useful in cartographic design decisions. Our previous own empirical work (Brychtová and Çöltekin 2015; Brychtová and Çöltekin 2016) has indeed confirmed the reliability of the ΔE_{00} metric, because we observed a clear tenden-

$$C_{1,ab} = \sqrt{(a_1)^2 + (b_1)^2} \quad C_{2,ab} = \sqrt{(a_2)^2 + (b_2)^2}$$

$$\bar{C}_{ab} = \frac{C_{1,ab} + C_{2,ab}}{2}$$

$$G = 0,5 \left(1 - \sqrt{\frac{\bar{C}_{ab}^7}{\bar{C}_{ab}^7 + 25^7}} \right)$$

$$a'_1 = (1 + G)a_1 \quad a'_2 = (1 + G)a_2$$

$$C'_1 = \sqrt{(a'_1)^2 + (b_1)^2} \quad C'_2 = \sqrt{(a'_2)^2 + (b_2)^2}$$

$$h'_1 = atan2(b_1, a'_1) = \begin{cases} \tan^{-1}\left(\frac{b_1}{a'_1}\right), & b_1 > 0, a'_1 \geq 0 \\ \tan^{-1}\left(\frac{b_1}{a'_1}\right) + 180^\circ, & b_1 < 0 \\ \tan^{-1}\left(\frac{b_1}{a'_1}\right) + 360^\circ, & b_1 > 0, a'_1 < 0 \\ 90^\circ, & b_1 = 0, a'_1 > 0 \\ 270^\circ, & b_1 = 0, a'_1 < 0 \\ 0^\circ, & b_1 = 0, a'_1 = 0 \end{cases}$$

$$h'_2 = atan2(b_2, a'_2) = \begin{cases} \tan^{-1}\left(\frac{b_2}{a'_2}\right), & b_2 > 0, a'_2 \geq 0 \\ \tan^{-1}\left(\frac{b_2}{a'_2}\right) + 180^\circ, & b_2 < 0 \\ \tan^{-1}\left(\frac{b_2}{a'_2}\right) + 360^\circ, & b_2 > 0, a'_2 < 0 \\ 90^\circ, & b_2 = 0, a'_2 > 0 \\ 270^\circ, & b_2 = 0, a'_2 < 0 \\ 0^\circ, & b_2 = 0, a'_2 = 0 \end{cases}$$

$$\Delta L' = L_2 - L_1$$

$$\Delta C' = C'_2 - C'_1$$

$$\Delta h' = \begin{cases} 0, & C'_1 C'_2 = 0 \\ h'_2 - h'_1, & C'_1 C'_2 \neq 0, |h'_2 - h'_1| \leq 180^\circ \\ (h'_2 - h'_1) - 360^\circ, & C'_1 C'_2 \neq 0, (h'_2 - h'_1) > 180^\circ \\ (h'_2 - h'_1) + 360^\circ, & C'_1 C'_2 \neq 0, (h'_2 - h'_1) < -180^\circ \end{cases}$$

$$\Delta H' = 2\sqrt{C'_1 C'_2} \sin\left(\frac{\Delta h'}{2}\right)$$

$$\bar{L}' = \frac{L_1 + L_2}{2}$$

$$\bar{C}' = \frac{C'_1 + C'_2}{2}$$

$$\bar{h}' = \begin{cases} \frac{h'_1 + h'_2}{2}, & C'_1 C'_2 \neq 0 \wedge |h'_1 - h'_2| \leq 180^\circ \\ \frac{h'_1 + h'_2 + 360^\circ}{2}, & |h'_1 - h'_2| > 180^\circ \wedge (h'_1 + h'_2) < 180^\circ \\ \frac{h'_1 + h'_2 - 360^\circ}{2}, & |h'_1 - h'_2| > 180^\circ \wedge (h'_1 + h'_2) \geq 360^\circ \\ h'_1 + h'_2, & C'_1 C'_2 = 0 \end{cases}$$

$$T = 1 - 0,17 \cos(\bar{h}' - 30^\circ) + 0,24 \cos(2\bar{h}') + 0,32 \cos(3\bar{h}' + 6^\circ) -$$

$$0,2 \cos(4\bar{h}' - 63^\circ)$$

$$\Delta\theta = 30e^{-\left(\frac{\bar{h}' - 275^\circ}{25}\right)^2}$$

$$R_C = 2 \sqrt{\frac{\bar{C}'^7}{\bar{C}'^7 + 25^7}}$$

$$S_L = 1 + \frac{0,015(\bar{L}' - 50)^2}{\sqrt{20 + (\bar{L}' - 50)^2}}$$

$$S_C = 1 + 0,0045\bar{C}'$$

$$S_H = 1 + 0,015\bar{C}'T$$

$$R_T = -\sin(2\Delta\theta)R_C$$

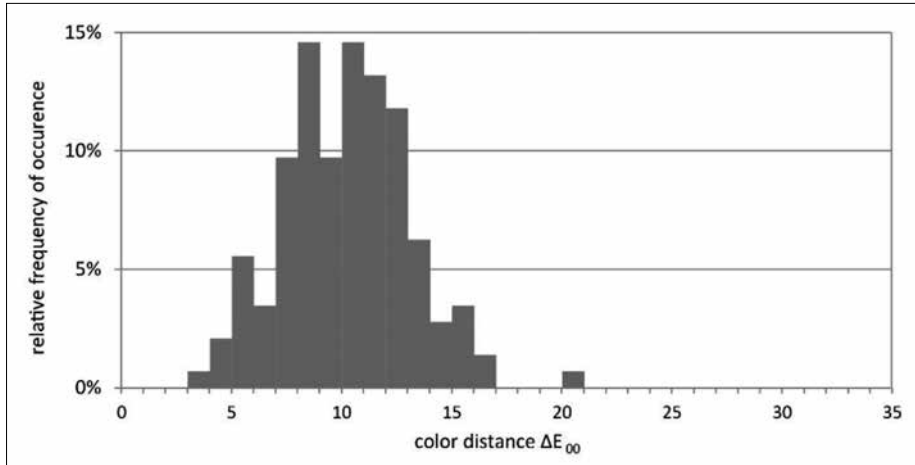


Fig. 5: Relative frequency of colour distance ΔE_{00} occurrence on 18 evaluated ColorBrewer 2.0 sequential colour schemes with 9 classes

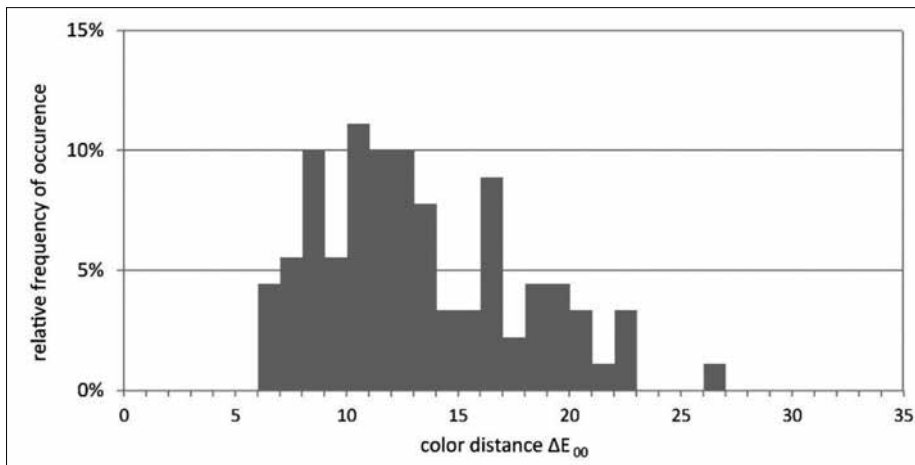


Fig. 6: Relative frequency of colour distance ΔE_{00} occurrence on 18 evaluated ColorBrewer 2.0 sequential colour schemes with 6 classes

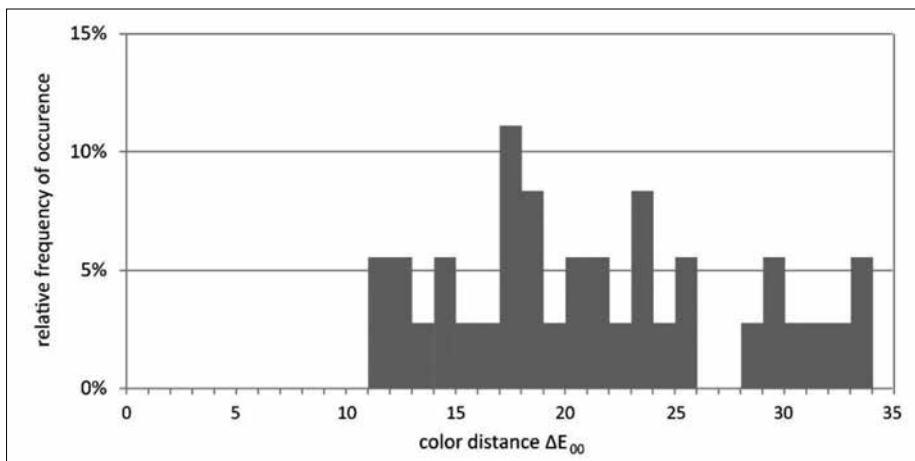


Fig. 7: Relative frequency of colour distance ΔE_{00} occurrence on 18 evaluated ColorBrewer 2.0 sequential colour schemes with 3 classes

cy that increasing ΔE_{00} values consistently improves the human judgement of whether two colours are the same or different both with sequential and qualitative schemes. Furthermore, as stated earlier, we observed that colour distance $\Delta E_{00} = 10$ 'works' in terms of same/different judgements for two colours, even if the compared colours are far

apart on the map. Using this knowledge from previous work of others as well as our previous own work, we presented an evaluation of a subset of colours used in the online software tool (colour recommender) *ColorBrewer 2.0* against $\Delta E_{00} = 10$ threshold as a minimum perceptually safe colour distance.

Our experimental results confirm that the *ColorBrewer 2.0* sequential colour schemes overall recommend distinguishable colours based on the ΔE_{00} metric and $\Delta E_{00} = 10$ threshold. However, there are some cases where the colour distance might not be ideal, i.e., the minimum colour distance in *ColorBrewer 2.0* was $\Delta E_{00, min} = 3.04$ (in one of the 9-class colour schemes), which was hardly distinguishable in our user experiments (Brychtová and Çöltekin 2016; Brychtová and Çöltekin 2015; Brychtová and Çöltekin 2014; Brychtová 2014; Brychtová and Vondráková 2014). Such small colour distance can potentially cause troubles for map reading. Furthermore, the distribution of colour distances within individual colour schemes of *ColorBrewer 2.0* was not constant. While this is expected to some degree (given the way Munsell colour space is organized), we observed that the colour distances are smaller between lighter shades, while they increase towards the darker shades. In our experiments (see Brychtová 2015), the best distribution of colour distances within a colour scheme was found to be increasing from both lightest and darkest shades towards the middle of the colour scheme (concave distribution, for schemes of six classes namely $\Delta E_{00} = 4-8-10-8-4$). More specifically, with this 'concave' distribution, we observed that the accuracy of the participants while matching the colour of mapped area with the legend was 93 %; while with colour schemes in which colour distance increased from the lightest to darkest shades (as it is in *ColorBrewer 2.0*), there was a 10 % loss in accuracy (83 % success). While 83 % success is also relatively high, if we can improve the accuracy by 10 % in colour matching tasks for map reading, we believe this must be considered. A dedicated set of user experiments to further confirm these observations would be a most beneficial next step.

At this point in time, colour perception is not fully understood. However, it is very clear that cartography as well as information visualization and scientific visualization communities would all benefit from a better-informed use of colour in designing displays. Required understanding varies from understanding basics of how colour is modelled; what can hardware (cameras, displays, devices) and software (browsers, your favorite graphics design tool) can/does render; and last but not least, what can humans process. We know, for example, that rainbow colours (even though they are used as the 'default' palette in many visualization

software) do not facilitate understanding patterns (Borland and Taylor 2007), and there is a spectrum of different colour deficiencies with humans (Jenny and Kelso 2007). We also understand when to use sequential and when to use diverging colours (Brewer 1996). There is, however, no absolute certainty on the minimum distinguishable difference between two colours/shades for the 'average' (healthy) human vision. We hope that, with this paper, we contribute further to the last point, and provide knowledge that might lead to more readable maps.

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