Statistical Natural Language Processing

N-gram Language Models

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N-gram language models

- A language model answers the question *how likely is a sequence of words in a given language?*
- They assign scores, typically probabilities, to sequences (of words, letters, …)
- n-gram language models are the 'classical' approach to language modeling
- The main idea is to estimate probabilities of sequences, using the probabilities of words given a limited history
- As a bonus we get the answer for *what is the most likely word given previous words?*

N-grams in practice: spelling correction

- How would a spell checker know that there is a spelling error in the following sentence?

N-grams in practice: spelling correction

- How would a spell checker know that there is a spelling error in the following sentence?

  I like pizza wit spinach

N-grams in practice: spelling correction

- How would a spell checker know that there is a spelling error in the following sentence?

  I like pizza wit spinach

- Or this one?

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- How would a spell checker know that there is a spelling error in the following sentence?

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- Or this one?

  Zoo animals on the lose

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N-grams in practice: spelling correction

- How would a spell checker know that there is a spelling error in the following sentence?

  I like pizza wit spinach

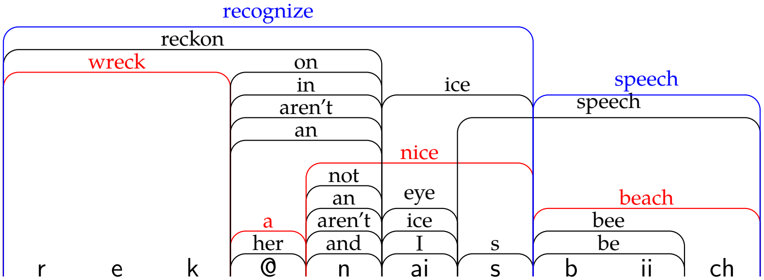
- Or this one?

  Zoo animals on the lose

We want:

$$
\begin{aligned}
P(\text{I like pizza with spinach}) &> P(\text{I like pizza wit spinach})\\
P(\text{Zoo animals on the loose}) &> P(\text{Zoo animals on the lose})
\end{aligned}
$$

N-grams in practice: speech recognition

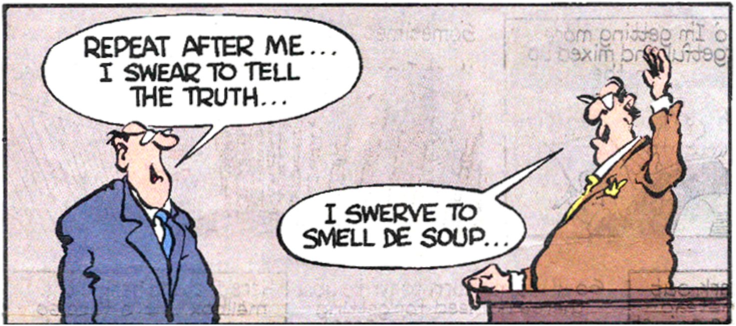


We want:

$$P(\text{recognize speech}) > P(\text{wreck a nice beach})$$

* Reproduced from Shillcock (1995)

Speech recognition gone wrong



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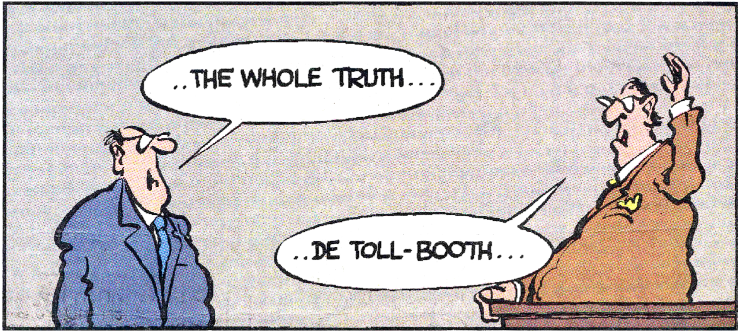

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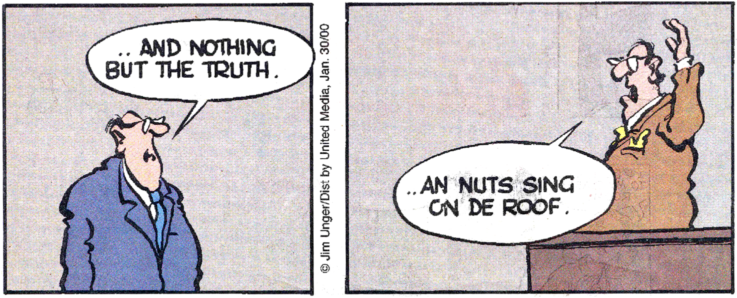
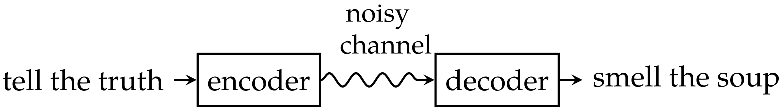


Speech recognition gone wrong

What went wrong?

Recap: noisy channel model



- We want $P(u \mid A)$, probability of the utterance given the acoustic signal
- From the noisy channel, we can get $P(A \mid u)$
- We can use Bayes' formula

$$P(u \mid A) = \frac{P(A \mid u)P(u)}{P(A)}$$

- $P(u)$, probabilities of utterances, come from a language model

N-grams in practice: machine translation

German to English translation:

- Correct word choice

German	English
Der grosse Mann tanzt gerne	*The big man likes to dance*
Der grosse Mann weiß alles	*The great man knows all*

- Correct ordering / word choice

German	English alternatives
Er tanzt gerne	*He dances with pleasure*
	He likes to dance

We want:

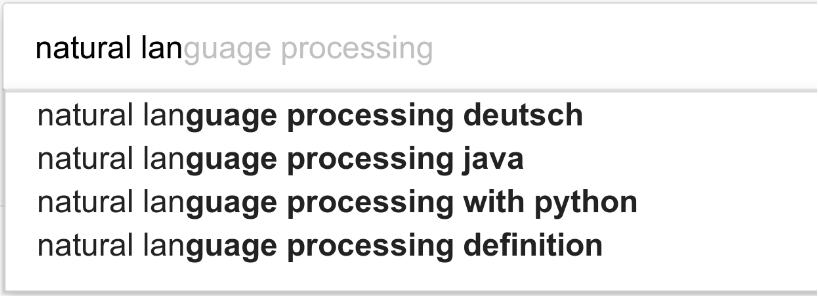
$$\mathrm{P}(\text{He likes to dance}) > \mathrm{P}(\text{He dances with pleasure})$$

N-grams in practice: predictive text

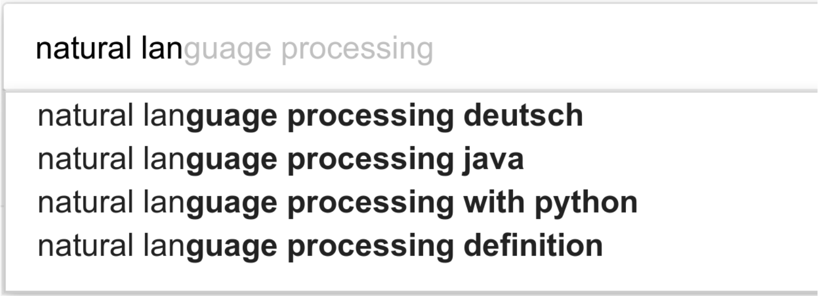
natural

natural **mojo**
natural**ismus**
natural **selection**
natural

N-grams in practice: predictive text



N-grams in practice: predictive text



- How many language models are there in the example above?
- Screenshot from google.com - but predictive text is used everywhere
- If you want examples of predictive text gone wrong, look for 'auto-correct mistakes' on the Web.

More applications for language models

- Spelling correction
- Speech recognition
- Machine translation
- Predictive text
- Text recognition (OCR, handwritten)
- Information retrieval
- Question answering
- Text classification
- …

Overview

of the overview

Why do we need n-gram language models?

What are they?

How do we build and use them?

What alternatives are out there?

Overview

in a bit more detail

- Why do we need n-gram language models?
- How to assign probabilities to sequences?
- N-grams: what are they, how do we count them?
- MLE: how to assign probabilities to n-grams?
- Evaluation: how do we know our n-gram model works well?
- Smoothing: how to handle unknown words?
- Some practical issues with implementing n-grams
- Extensions, alternative approaches

Our aim

We want to solve two related problems:

- Given a sequence of words $\boldsymbol{w} = (w_1 w_2 \dots w_m)$,

  what is the probability of the sequence $P(\boldsymbol{w})$?

  (machine translation, automatic speech recognition, spelling correction)

- Given a sequence of words $w_1 w_2 \dots w_{m-1}$,

  what is the probability of the next word $P(w_m \mid w_1 \dots w_{m-1})$?

  (predictive text)

Assigning probabilities to sentences

count and divide?

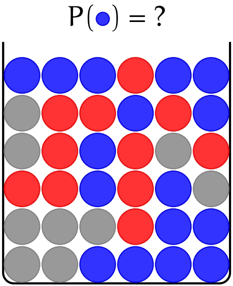
How do we calculate the probability a sentence like $P(\text{I like pizza wit spinach})$

Assigning probabilities to sentences

count and divide?

How do we calculate the probability a sentence like P(I like pizza wit spinach)

- Can we count the occurrences of the sentence, and divide it by the total number of sentences (in a large corpus)?



Assigning probabilities to sentences

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How do we calculate the probability a sentence like P(I like pizza wit spinach)

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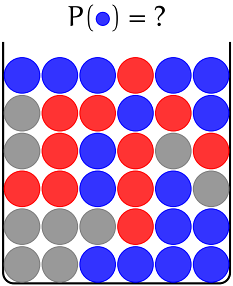
- Short answer: No.



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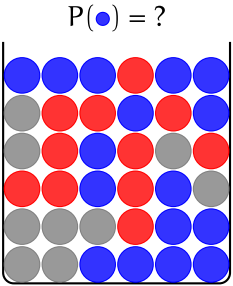
- Short answer: No.
  - Many sentences are not observed even in very large corpora



Assigning probabilities to sentences

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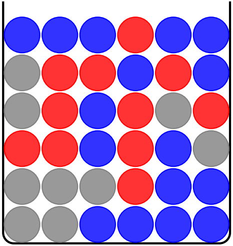
How do we calculate the probability a sentence like P(I like pizza wit spinach)

- Can we count the occurrences of the sentence, and divide it by the total number of sentences (in a large corpus)?
- Short answer: No.
  - Many sentences are not observed even in very large corpora
  - For the ones observed in a corpus, probabilities will not reflect our intuition, or will not be useful in most applications

P(●) = ?



Assigning probabilities to sentences

applying the chain rule

- The solution is to *decompose*

  We use probabilities of parts of the sentence (words) to calculate the probability of the whole sentence

- Using the chain rule of probability (without loss of generality), we can write

$$
\begin{aligned}
P(w_1, w_2, \ldots, w_m) = \; & P(w_2 \mid w_1) \\
& \times P(w_3 \mid w_1, w_2) \\
& \times \ldots \\
& \times P(w_m \mid w_1, w_2, \ldots w_{m-1})
\end{aligned}
$$

Example: applying the chain rule

$$
\begin{aligned}
P(\text{I like pizza with spinach}) = \;& P(\text{like} \mid \text{I}) \\
& \times P(\text{pizza} \mid \text{I like}) \\
& \times P(\text{with} \mid \text{I like pizza}) \\
& \times P(\text{spinach} \mid \text{I like pizza with})
\end{aligned}
$$

- Did we solve the problem?

Example: applying the chain rule

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P(\text{I like pizza with spinach}) = \; & P(\text{like} \mid \text{I}) \\
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& \times P(\text{spinach} \mid \text{I like pizza with})
\end{aligned}
$$

- Did we solve the problem?
- Not really, the last term is equally difficult to estimate

Assigning probabilities to sentences

the Markov assumption

We make a *conditional independence* assumption: *probabilities of words are independent, given* n *previous words*

$$P(w_i \mid w_1, \ldots, w_{i-1}) = P(w_i \mid w_{i-n+1}, \ldots, w_{i-1})$$

and

$$P(w_1, \ldots, w_m) = \prod_{i=1}^{m} P(w_i \mid w_{i-n+1}, \ldots, w_{i-1})$$

For example, with $n = 2$ (bigram, first order Markov model):

$$P(w_1, \ldots, w_m) = \prod_{i=1}^{m} P(w_i \mid w_{i-1})$$

Example: bigram probabilities of a sentence

$$
\begin{aligned}
P(\text{I like pizza with spinach}) = \; & P(\text{like} \mid \text{I}) \\
& \times P(\text{pizza} \mid \text{I like}) \\
& \times P(\text{with} \mid \text{I like pizza}) \\
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\end{aligned}
$$

Example: bigram probabilities of a sentence

$$
\begin{aligned}
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& \times P(\text{pizza} \mid \text{like}) \\
& \times P(\text{with} \mid \text{pizza}) \\
& \times P(\text{spinach} \mid \text{with})
\end{aligned}
$$

- Now, hopefully, we can count them in a corpus

Maximum-likelihood estimation (MLE)

- Maximum-likelihood estimation of n-gram probabilities is based on their frequencies in a corpus
- We are interested in conditional probabilities of the form: $P(w_i \mid w_1, \ldots, w_{i-1})$, which we estimate using

$$P(w_i \mid w_{i-n+1}, \ldots, w_{i-1}) = \frac{C(w_{i-n+1} \ldots w_i)}{C(w_{i-n+1} \ldots w_{i-1})}$$

  where, $C()$ is the frequency (count) of the sequence in the corpus.
- For example, the probability $P(\text{like} \mid \text{I})$ would be

$$\begin{aligned} P(\text{like} \mid \text{I}) &= \frac{C(\text{I like})}{C(\text{I})} \\ &= \frac{\text{number of times I like occurs in the corpus}}{\text{number of times I occurs in the corpus}} \end{aligned}$$

MLE estimation of an n-gram language model

An n-gram model conditioned on $n-1$ previous words.

- In a 1-gram (unigram) model,

$$P(w_i) = \frac{C(w_i)}{N}$$

- In a 2-gram (bigram) model,

$$P(w_i) = P(w_i \mid w_{i-1}) = \frac{C(w_{i-1}w_i)}{C(w_{i-1})}$$

- In a 3-gram (trigram) model,

$$P(w_i) = P(w_i \mid w_{i-2}w_{i-1}) = \frac{C(w_{i-2}w_{i-1}w_i)}{C(w_{i-2}w_{i-1})}$$

Training an n-gram model involves estimating these parameters (conditional probabilities).

Unigrams

Unigrams are simply the single words (or tokens).

A small corpus

I'm sorry, Dave.
I'm afraid I can't do that.

Unigrams

Unigrams are simply the single words (or tokens).

A small corpus

I ’m sorry , Dave .
I ’m afraid I can ’t do that .

When tokenized, we have 15 *tokens*, and 11 *types*.

Unigram counts

ngram	freq	ngram	freq	ngram	freq	ngram	freq
I	3	,	1	afraid	1	do	1
’m	2	Dave	1	can	1	that	1
sorry	1	.	2	’t	1		

Traditionally, *can’t* is tokenized as *ca␣n’t* (similar to *have␣n’t*, *is␣n’t* etc.), but for our purposes *can␣’t* is more readable.

Unigram probability of a sentence

Unigram counts

ngram	freq	ngram	freq	ngram	freq	ngram	freq
I	3	,	1	afraid	1	do	1
'm	2	Dave	1	can	1	that	1
sorry	1	.	2	't	1		

$$
\begin{aligned}
&\mathtt{P(I\ 'm\ sorry\ ,\ Dave\ .)}\\
&= \mathtt{P(I)} \times \mathtt{P('m)} \times \mathtt{P(sorry)} \times \mathtt{P(,)} \times \mathtt{P(Dave)} \times \mathtt{P(.)}\\
&= \frac{3}{15} \times \frac{2}{15} \times \frac{1}{15} \times \frac{1}{15} \times \frac{1}{15} \times \frac{2}{15}\\
&= 0.000\,001\,05
\end{aligned}
$$

Unigram probability of a sentence

Unigram counts

ngram	freq	ngram	freq	ngram	freq	ngram	freq
I	3	,	1	afraid	1	do	1
’m	2	Dave	1	can	1	that	1
sorry	1	.	2	’t	1		

$$
\begin{aligned}
&\texttt{P(I 'm sorry , Dave .)} \\
&= \texttt{P(I)} \times \texttt{P('m)} \times \texttt{P(sorry)} \times \texttt{P(,)} \times \texttt{P(Dave)} \times \texttt{P(.)} \\
&= \frac{3}{15} \times \frac{2}{15} \times \frac{1}{15} \times \frac{1}{15} \times \frac{1}{15} \times \frac{2}{15} \\
&= 0.000\,001\,05
\end{aligned}
$$

- $\texttt{P(, 'm I . sorry Dave)} = ?$
- What is the most likely sentence according to this model?

N-gram models define probability distributions

- An n-gram model defines a probability distribution over words

$$\sum_{w \in V} P(w) = 1$$

- They also define probability distributions over word sequences of equal size. For example (length 2),

$$\sum_{w \in V} \sum_{v \in V} P(w)P(v) = 1$$

word	prob
I	0.200
’m	0.133
.	0.133
’t	0.067
,	0.067
Dave	0.067
afraid	0.067
can	0.067
do	0.067
sorry	0.067
that	0.067
	1.000

N-gram models define probability distributions

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$$\sum_{w \in V} \sum_{v \in V} P(w)P(v) = 1$$

- What about sentences?

word	prob
I	0.200
'm	0.133
.	0.133
't	0.067
,	0.067
Dave	0.067
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do	0.067
sorry	0.067
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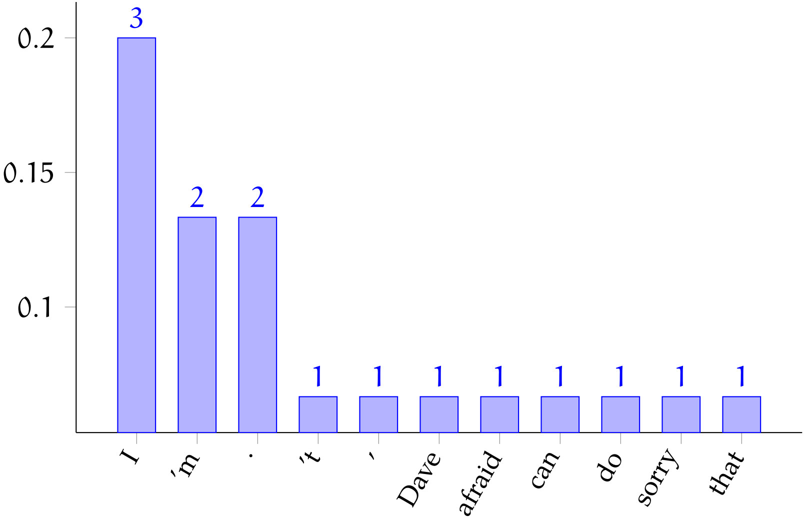
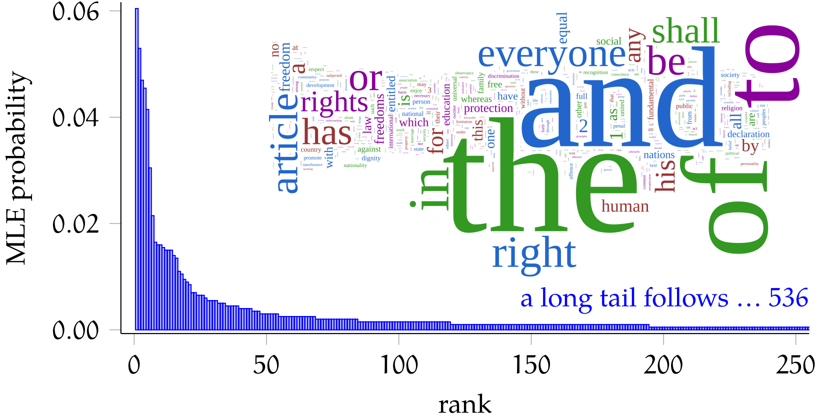
Unigram probabilities



Unigram probabilities in a (slightly) larger corpus

MLE probabilities in the Universal Declaration of Human Rights



Zipf's law – a short divergence

The frequency of a word is inversely proportional to its rank:

$$\text{rank} \times \text{frequency} = k \quad \text{or} \quad \text{frequency} \propto \frac{1}{\text{rank}}$$

- This is a reoccurring theme in (computational) linguistics: most linguistic units follow more-or-less a similar distribution
- Important consequence for us (in this lecture):
  - even very large corpora will *not* contain some of the words (or n-grams)

Bigrams

Bigrams are overlapping sequences of two tokens.

I ’m sorry , Dave .

I ’m afraid I can ’t do that .

Bigram counts

ngram	freq	ngram	freq	ngram	freq	ngram	freq
I ’m	2	, Dave	1	afraid I	1	n’t do	1
’m sorry	1	Dave .	1	I can	1	do that	1
sorry ,	1	’m afraid	1	can ’t	1	that .	1

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'm sorry	1	Dave .	1	I can	1	do that	1
sorry ,	1	'm afraid	1	can 't	1	that .	1

- What about the bigram '`. I`'?

Sentence boundary markers

If we want sentence probabilities, we need to mark them.

$\langle s \rangle$ I 'm sorry , Dave . $\langle /s \rangle$
$\langle s \rangle$ I 'm afraid I can 't do that . $\langle /s \rangle$

- The bigram ' $\langle$`s`$\rangle$ `I` ' is not the same as the unigram ' `I` '
  Including $\langle s \rangle$ allows us to predict likely words at the beginning of a sentence
- Including $\langle /s \rangle$ allows us to assign a proper probability distribution to sentences

Calculating bigram probabilities

recap with some more detail

We want to calculate $P(w_2 \mid w_1)$. From the chain rule:

$$P(w_2 \mid w_1) = \frac{P(w_1, w_2)}{P(w_1)}$$

and, the MLE

$$P(w_2 \mid w_1) = \frac{\frac{C(w_1 w_2)}{N}}{\frac{C(w_1)}{N}} = \frac{C(w_1 w_2)}{C(w_1)}$$

$P(w_2 \mid w_1)$ is the probability of w_2 given the previous word is w_1

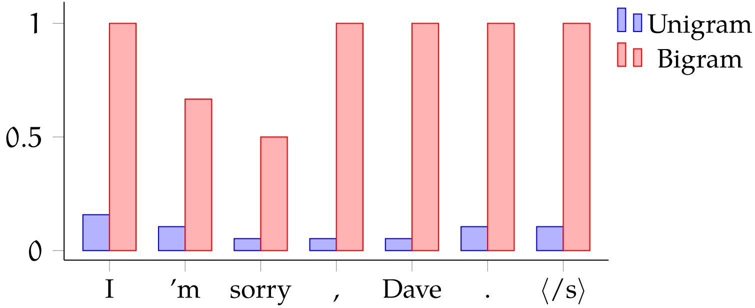
$P(w_2, w_1)$ is the probability of the sequence $w_1 w_2$

$P(w_1)$ is the probability of w_1 occurring as the first item in a bigram, not its unigram probability unigram probability!

Bigram probabilities

$w_1 w_2$	$C(w_1 w_2)$	$C(w_1)$	$P(w_1 w_2)$	$P(w_1)$	$P(w_2 \mid w_1)$	$P(w_2)$
⟨s⟩ I	2	2	0.12	0.12	1.00	0.18
I ’m	2	3	0.12	0.18	0.67	0.12
’m sorry	1	2	0.06	0.12	0.50	0.06
sorry ,	1	1	0.06	0.06	1.00	0.06
, Dave	1	1	0.06	0.06	1.00	0.06
Dave .	1	1	0.06	0.06	1.00	0.12
’m afraid	1	2	0.06	0.12	0.50	0.06
afraid I	1	1	0.06	0.06	1.00	0.18
I can	1	3	0.06	0.18	0.33	0.06
can ’t	1	1	0.06	0.06	1.00	0.06
n’t do	1	1	0.06	0.06	1.00	0.06
do that	1	1	0.06	0.06	1.00	0.06
that .	1	1	0.06	0.06	1.00	0.12
. ⟨/s⟩	2	2	0.12	0.12	1.00	0.12

Sentence probability: bigram vs. unigram



$$P_{uni}(\langle s \rangle \text{ I 'm sorry , Dave . } \langle /s \rangle) = 2.83 \times 10^{-9}$$

$$P_{bi}(\langle s \rangle \text{ I 'm sorry , Dave . } \langle /s \rangle) = 0.33$$

Unigram vs. bigram probabilities

in sentences and non-sentences

w	I	'm	sorry	,	Dave	.	
P_{uni}	0.20	0.13	0.07	0.07	0.07	0.07	2.83×10^{-9}
P_{bi}	1.00	0.67	0.50	1.00	1.00	1.00	0.33

Unigram vs. bigram probabilities

in sentences and non-sentences

w	I	'm	sorry	,	Dave	.	
P_{uni}	0.20	0.13	0.07	0.07	0.07	0.07	2.83×10^{-9}
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w	,	'm	I	.	sorry	Dave	
P_{uni}	0.07	0.13	0.20	0.07	0.07	0.07	2.83×10^{-9}
P_{bi}	0.00	0.00	0.00	0.00	0.00	1.00	0.00

Unigram vs. bigram probabilities

in sentences and non-sentences

w	I	'm	sorry	,	Dave	.	
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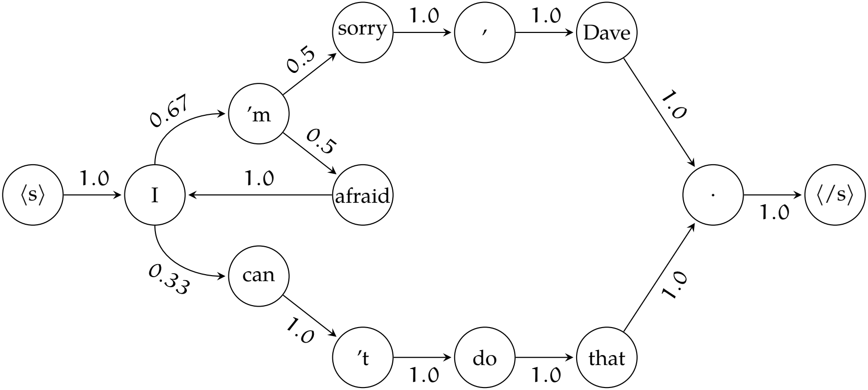
P_{bi}	1.00	0.67	0.50	1.00	1.00	1.00	0.33

w	,	'm	I	.	sorry	Dave	
P_{uni}	0.07	0.13	0.20	0.07	0.07	0.07	2.83×10^{-9}
P_{bi}	0.00	0.00	0.00	0.00	0.00	1.00	0.00

w	I	'm	afraid	,	Dave	.	
P_{uni}	0.07	0.13	0.07	0.07	0.07	0.07	2.83×10^{-9}
P_{bi}	1.00	0.67	0.50	0.00	0.50	1.00	0.00

Bigram model as a finite-state automaton



Trigrams

⟨s⟩ ⟨s⟩ I 'm sorry , Dave . ⟨/s⟩
⟨s⟩ ⟨s⟩ I 'm afraid I can 't do that . ⟨/s⟩

Trigram counts

ngram	freq	ngram	freq	ngram	freq
⟨s⟩ ⟨s⟩ I	2	do that .	1	that . ⟨/s⟩	1
⟨s⟩ I 'm	2	I 'm sorry	1	'm sorry ,	1
sorry , Dave	1	, Dave .	1	Dave . ⟨/s⟩	1
I 'm afraid	1	'm afraid I	1	afraid I can	1
I can 't	1	can 't do	1	't do that	1

Trigrams

⟨s⟩ ⟨s⟩ I 'm sorry , Dave . ⟨/s⟩
⟨s⟩ ⟨s⟩ I 'm afraid I can 't do that . ⟨/s⟩

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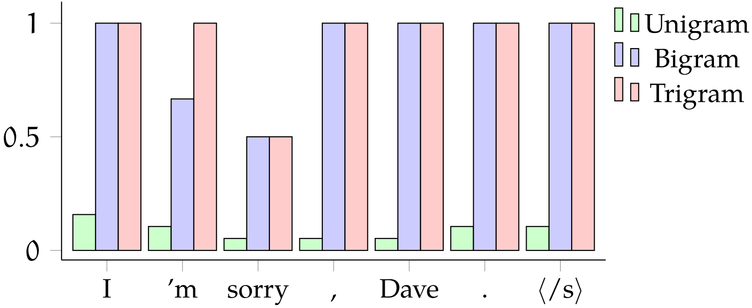
⟨s⟩ I 'm	2	I 'm sorry	1	'm sorry ,	1
sorry , Dave	1	, Dave .	1	Dave . ⟨/s⟩	1
I 'm afraid	1	'm afraid I	1	afraid I can	1
I can 't	1	can 't do	1	't do that	1

- How many n-grams are there in a sentence of length m?

Trigram probabilities of a sentence



$$
\begin{aligned}
P_{\text{uni}}(\text{I 'm sorry , Dave . } \langle/\text{s}\rangle) &= 2.83 \times 10^{-9} \\
P_{\text{bi}}(\text{I 'm sorry , Dave . } \langle/\text{s}\rangle) &= 0.33 \\
P_{\text{tri}}(\text{I 'm sorry , Dave . } \langle/\text{s}\rangle) &= 0.50
\end{aligned}
$$

Short detour: colorless green ideas

But it must be recognized that the notion 'probability of a sentence' is an entirely useless one, under any known interpretation of this term. — Chomsky (1968)

- The following 'sentences' are categorically different:
  - Furiously sleep ideas green colorless
  - Colorless green ideas sleep furiously

Short detour: colorless green ideas

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- The following 'sentences' are categorically different:
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- Can n-gram models model the difference?
- Should n-gram models model the difference?

What do n-gram models model?

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N-gram models are practical tools, and they have been useful for many tasks.

N-grams, so far …

- N-gram language models are one of the basic tools in NLP
- They capture some linguistic (and non-linguistic) regularities that are useful in many applications
- The idea is to estimate the probability of a sentence based on its parts (sequences of *words*)
- N-grams are n consecutive units in a sequence
- Typically, we use sequences of *words* to estimate sentence probabilities, but other units are also possible: *characters*, *phonemes*, *phrases*, …
- For most applications, we introduce sentence boundary markers

N-grams, so far …

- The most straightforward method for estimating probabilities is using relative frequencies (leads to MLE)
- Due to Zipf's law, as we increase 'n', the counts become smaller (data sparseness), many counts become 0
- If there are unknown words, we get 0 probabilities for both words and sentences
- In practice, bigrams or trigrams are used most commonly, applications/datasets of up to 5-grams are also used

How to test n-gram models?

Extrinsic: how (much) the model improves the target application:

- Speech recognition accuracy
- BLEU score for machine translation
- Keystroke savings in predictive text applications

Intrinsic: the higher the probability assigned to a test set better the model. A few measures:

- Likelihood
- (cross) entropy
- perplexity

Training and test set division

- We (almost) never use a statistical (language) model on the training data
- Testing a model on the training set is misleading: the model may overfit the training set
- Always test your models on a separate *test set*

Intrinsic evaluation metrics: likelihood

- Likelihood of a model M is the probability of the (test) set $\boldsymbol{w}$ given the model

$$\mathcal{L}(M \mid \boldsymbol{w}) = P(\boldsymbol{w} \mid M) = \prod_{s \in w} P(s)$$

- The higher the likelihood (for a given test set), the better the model
- Likelihood is sensitive to test set size
- Practical note: (minus) log likelihood is more common, because of readability and ease of numerical manipulation

Intrinsic evaluation metrics: cross entropy

- Cross entropy of a language model on a test set $\boldsymbol{w}$ is

$$H(\boldsymbol{w}) = -\frac{1}{N}\log_2 P(\boldsymbol{w})$$

- The lower the cross entropy, the better the model
- Remember that cross entropy is the average bits required to encode the data coming from a distribution (test set distribution) using an approximate distribution (the language model)
- Note that cross entropy is not sensitive to length of the test set

Intrinsic evaluation metrics: perplexity

- Perplexity is a more common measure for evaluating language models

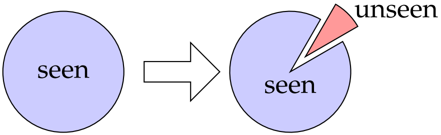
$$\mathrm{PP}(\boldsymbol{w}) = 2^{\mathrm{H}(w)} = \mathrm{P}(\boldsymbol{w})^{-\frac{1}{N}} = \sqrt[N]{\frac{1}{\mathrm{P}(\boldsymbol{w})}}$$

- Perplexity is the average branching factor
- Similar to cross entropy
  - lower better
  - not sensitive to test set size

What do we do with unseen n-grams?

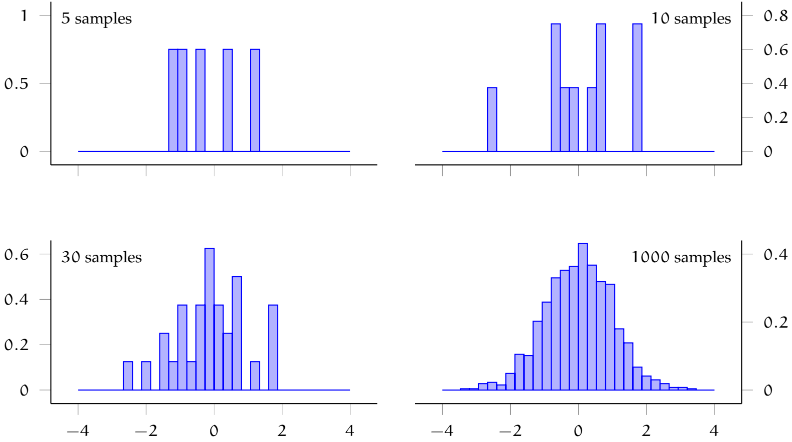
and other issues with MLE estimates

- Words (and word sequences) are distributed according to the Zipf's law: *many words are rare*.
- MLE will assign 0 probabilities to unseen words, and sequences containing unseen words
- Even with non-zero probabilities, MLE *overfits* the training data
- One solution is smoothing: take some probability mass from known words, and assign it to unknown words



Smoothing: what is in the name?

samples from $\mathcal{N}(0, 1)$



Laplace smoothing

(Add-one smoothing)

- The idea (from 1790): add one to all counts
- The probability of a word is estimated by

$$P_{+1}(w) = \frac{C(w)+1}{N+V}$$

N number of word tokens
V number of word types - the size of the vocabulary

- Then, probability of an unknown word is:

$$\frac{0+1}{N+V}$$

Laplace smoothing

for n-grams

- The probability of a bigram becomes

$$P_{+1}(w_i w_{i-1}) = \frac{C(w_i w_{i-1}) + 1}{N + V^2}$$

- and, the conditional probability

$$P_{+1}(w_i \mid w_{i-1}) = \frac{C(w_{i-1} w_i) + 1}{C(w_{i-1}) + V}$$

- In general

$$P_{+1}(w_{i-n+1}^{i}) = \frac{C(w_{i-n+1}^{i}) + 1}{N + V^n}$$

$$P_{+1}(w_{i-n+1}^{i} \mid w_{i-n+1}^{i-1}) = \frac{C(w_{i-n+1}^{i}) + 1}{C(w_{i-n+1}^{i-1}) + V}$$

Bigram probabilities

non-smoothed vs. Laplace smoothing

$w_1 w_2$	C_{+1}	$P_{MLE}(w_1 w_2)$	$P_{+1}(w_1 w_2)$	$P_{MLE}(w_2 \mid w_1)$	$P_{+1}(w_2 \mid w_1)$
⟨s⟩ I	3	0.118	0.019	1.000	0.188
I 'm	3	0.118	0.019	0.667	0.176
'm sorry	2	0.059	0.012	0.500	0.125
sorry ,	2	0.059	0.012	1.000	0.133
, Dave	2	0.059	0.012	1.000	0.133
Dave .	2	0.059	0.012	1.000	0.133
'm afraid	2	0.059	0.012	0.500	0.125
afraid I	2	0.059	0.012	1.000	0.133
I can	2	0.059	0.012	0.333	0.118
can 't	2	0.059	0.012	1.000	0.133
n't do	2	0.059	0.012	1.000	0.133
do that	2	0.059	0.012	1.000	0.133
that .	2	0.059	0.012	1.000	0.133
. ⟨/s⟩	3	0.118	0.019	1.000	0.188
$\sum$		1.000	0.193		

MLE vs. Laplace probabilities

bigram probabilities in sentences and non-sentences

w	I	'm	sorry	,	Dave	.	⟨/s⟩	
P_{MLE}	1.00	0.67	0.50	1.00	1.00	1.00	1.00	0.33
P_{+1}	0.25	0.23	0.17	0.18	0.18	0.18	0.25	1.44×10^{-5}

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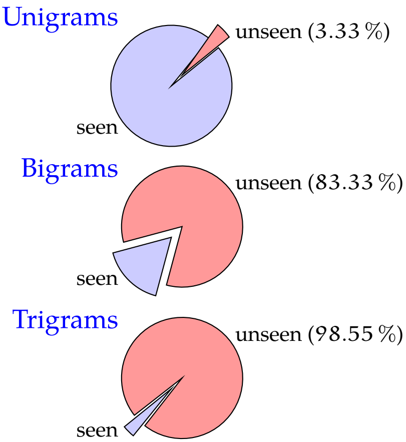
w	I	'm	afraid	,	Dave	.	⟨/s⟩	
P_{uni}	1.00	0.67	0.50	0.00	1.00	1.00	1.00	0.00
P_{bi}	0.25	0.23	0.17	0.09	0.18	0.18	0.25	7.22×10^{-6}

How much mass does +1 smoothing steal?

- Laplace smoothing reserves probability mass proportional to vocabulary size of the vocabulary
- This is just too much for large vocabularies and higher order n-grams
- Note that only very few of the higher level n-grams (e.g., trigrams) are possible



Lindstone correction

(Add-α smoothing)

- A simple improvement over Laplace smoothing is adding $0 < \alpha$ (and typically < 1) instead of 1

$$P(w_i^{i-n+1}) = \frac{C(w_i^{i-n+1}) + \alpha}{N + \alpha V}$$

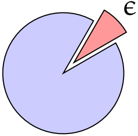
- With smaller α values, the model behaves similar to MLE, it has high variance: it overfits
- Larger α values reduce the variance, but has large bias

How do we pick a good α value

setting smoothing parameters

- We want α value that works best outside the training data
- Peeking at your test data during training/development is wrong
- This calls for another division of the available data: set aside a *development set* for tuning *hyperparameters*
- Alternatively, we can use k-fold cross validation and take the α with the best average score (more on cross validation later in this course)

Absolute discounting



- An alternative to the additive smoothing is to reserve an explicit amount of probability mass, ϵ, for the unseen events
- The probabilities of known events has to be re-normalized
- This is often not very convenient
- How do we decide what ϵ value to use?

Good-Turing smoothing

'discounting' view

- Estimate the probability mass to be reserved for the novel n-grams using the observed n-grams
- Novel events in our training set is the ones that occur once

$$p_0 = \frac{n_1}{n}$$

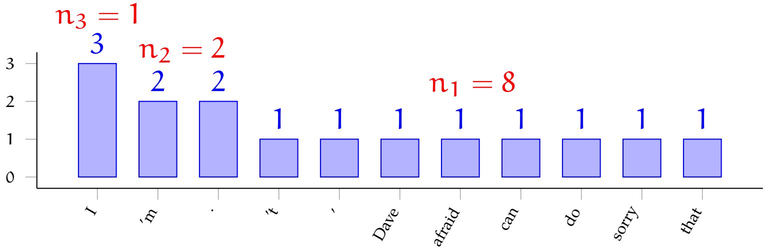
where n_1 is the number of distinct n-grams with frequency 1 in the training data

- Now we need to discount this mass from the higher counts
- The probability of an n-gram that occurred r times in the data corpus is

$$(r+1)\frac{n_{r+1}}{n_r n}$$

Some terminology

frequencies of frequencies and equivalence classes



- We often put n-grams into equivalence classes
- Good-Turing forms the equivalence classes based on frequency

Note:

$$n = \sum_r r \times n_r$$

Good-Turing estimation: leave-one-out justification

- Leave each n-gram out
- Count the number of times the left-out n-gram had frequency r in the remaining data
  - novel n-grams

$$\frac{n_1}{n}$$

  - n-grams with frequency 1 (singletons)

$$(1+1)\frac{n_2}{n_1 n}$$

  - n-grams with freqnency 2 (doubletons)*

$$(2+1)\frac{n_3}{n_2 n}$$

* Yes, this seems to be a word.

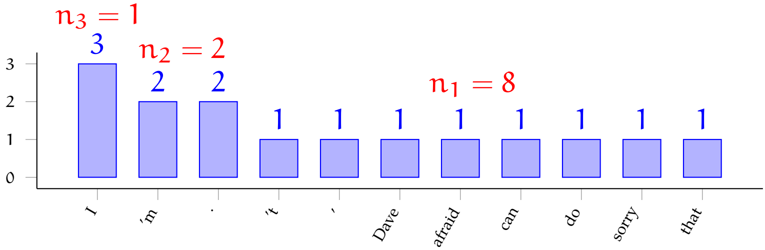
Adjusted counts

Sometimes it is instructive to see the 'effective count' of an n-gram under the smoothing method.
For Good-Turing smoothing, the updated count, r^* is

$$r^* = (r+1)\frac{n_{r+1}}{n_r}$$

- novel items: n_1
- singeltons: $\frac{2 \times n_2}{n_1}$
- doubletons: $\frac{3 \times n_3}{n_2}$
- ...

Good-Turing example



$$P_{GT}(\text{the}) = P_{GT}(\text{a}) = \ldots = \frac{8}{15}$$

$$P_{GT}(\text{that}) = P_{GT}(\text{do}) = \ldots = \frac{2 \times 2}{15}$$

$$P_{GT}(\text{'m}) = P_{GT}(.) = \frac{3 \times 1}{15}$$

Issues with Good-Turing discounting

With some solutions

- Zero counts: we cannot assign probabilities if $n_{r+1} = 0$
- The estimates of some of the frequencies of frequencies are unreliable
- A solution is to replace n_r with smoothed counts z_r
- A well-known technique (simple Good-Turing) for smoothing n_r is to use linear interpolation

$$\log z_r = a + b \log r$$

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N-grams, so far …

- Two different ways of evaluating n-gram models:

Extrinsic success in an external application

Intrinsic likelihood, (cross) entropy, perplexity

- Intrinsic evaluation metrics often correlate well with the extrinsic metrics
- Test your n-grams models on an 'unseen' test set

N-grams, so far …

- Smoothing methods solve the zero-count problem (also reduce the variance)
- Smoothing takes away some probability mass from the observed n-grams, and assigns it to unobserved ones
  - Additive smoothing: add a constant α to all counts
    - $\alpha = 1$ (Laplace smoothing) simply adds one to all counts – simple but often not very useful
    - A simple correction is to add a smaller α, which requires tuning over a development set
  - Discounting removes a fixed amount of probability mass, ϵ, from the observed n-grams
    - We need to re-normalize the probability estimates
    - Again, we need a development set to tune ϵ
  - Good-Turing discounting reserves the probability mass to the unobserved events based on the n-grams seen only once: $p_0 = \frac{n_1}{n}$

Not all (unknown) n-grams are equal

- Let's assume that `black squirrel` is an unknown bigram
- How do we calculate the smoothed probability

$$P_{+1}(\texttt{squirrel} \mid \texttt{black}) =$$

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- How about `black wug`?

$$P_{+1}(\texttt{black wug}) = P_{+1}(\texttt{squirrel} \mid \texttt{wug}) = \frac{0+1}{C(\texttt{black}) + V}$$

- Would make a difference if we used a better smoothing method (e.g., Good-Turing?)

Back-off and interpolation

The general idea is to fall-back to lower order n-gram when estimation is unreliable

- Even if,

$$C(\texttt{black squirrel}) = C(\texttt{black wug}) = 0$$

it is unlikely that

$$C(\texttt{squirrel}) = C(\texttt{wug})$$

in a reasonably sized corpus

Back-off

Back-off uses the estimate if it is available, 'backs off' to the lower order n-gram(s) otherwise:

$$P(w_i \mid w_{i-1}) = \begin{cases} P^*(w_i \mid w_{i-1}) & \text{if } C(w_{i-1}wi) > 0 \\ \alpha P(w_i) & \text{otherwise} \end{cases}$$

where,

- $P^*(\cdot)$ is the discounted probability
- α makes sure that $\sum P(w)$ is the discounted amount
- $P(w_i)$, typically, smoothed unigram probability

Interpolation

Interpolation uses a linear combination:

$$P_{int}(w_i \mid w_{i-1}) = \lambda P(w_i \mid w_{i-1}) + (1 - \lambda) P(w_i)$$

In general (recursive definition),

$$P_{\text{int}}(w_i \mid w_{i-n+1}^{i-1}) = \lambda P(w_i \mid w_{i-n+1}^{i-1}) + (1 - \lambda) P_{\text{int}}(w_i \mid w_{i-n+2}^{i-1})$$

- $\sum \lambda_i = 1$
- Recursion terminates
  - either smoothed unigram counts
  - or uniform distribution $\frac{1}{V}$

Not all contexts are equal

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  - `black squirrel`
  - `wreak squirrel`

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- Back to our example: given both bigrams
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  are unknown, the above formulations assign the same probability to both bigrams
- To solve this, the back-off or interpolation parameters (α or λ) are often conditioned on the context
- For example,

$$P_{int}(w_i \mid w_{i-n+1}^{i-1}) = \lambda_{w_{i-n+1}^{i-1}} P(w_i \mid w_{i-n+1}^{i-1}) + (1 - \lambda_{w_{i-n+1}^{i-1}}) P_{int}(w_i \mid w_{i-n+2}^{i-1})$$

Katz back-off

A popular back-off method is Katz back-off:

$$P_{Katz}(w_i|w_{i-n+1}^{i-1}) = \begin{cases} P^*(w_i \mid w_{i-n+1}^{i-1}) & \text{if } C(w_{i-n+1}^{i}) > 0 \\ \alpha_{w_{i-n+1}^{i-1}} P_{katz}(w_i \mid w_{i-n+2}^{i-1}) & \text{otherwise} \end{cases}$$

- $P^*(\cdot)$ is the Good-Turing discounted probability estimate (only for n-grams with small counts)
- $\alpha_{w_{i-n+1}^{i-1}}$ makes sure that the back-off probabilities sums to the discounted amount
- α is high for the unknown words that appear in frequent contexts

Kneser-Ney interpolation: intuition

- Use absolute discounting for the higher order n-gram
- Estimate the lower order n-gram probabilities based on the probability of the target word occurring in a new context
- Example:
  `I can't see without my reading _____.`

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- Example:
  `I can't see without my reading glasses.`
- It turns out `Francisco` is much more frequent than `glasses`
- But `Francisco` occurs only in the context `San Francisco`
- Assigning probabilities to unigrams based on the number of unique context they appear makes `glasses` more likely

Kneser-Ney interpolation

for bigrams

Absolute discount

Unique contexts w_i appears

$$P_{KN}(w_i|w_{i-1}) = \frac{C(w_{i-1}w_i) - D}{C(w_i)} + \lambda_{w_{i-1}} \frac{|\{v \mid C(vw_i) > 0\}|}{\sum_w |\{v \mid C(vw) > 0\}|}$$

All unique contexts

- λs make sure that the probabilities sum to 1
- The same idea can be applied to back-off as well (interpolation seems to work better)

Some shortcomings of the n-gram language models

The n-gram language models are simple and successful, but …

- They are highly sensitive to the training data: you do not want to use an n-gram model trained on business news for medical texts
- They cannot handle long-distance dependencies:
  `In the last race, the horse he bought last year finally _____.`
- The success often drops in morphologically complex languages
- The smoothing interpolation methods are often 'a bag of tricks'

Cluster-based n-grams

- The idea is to cluster the words, and fall-back (back-off or interpolate) to the cluster
- For example,
  - a clustering algorithm is likely to form a cluster containing words for food, e.g., `{apple, pear, broccoli, spinach}`
  - if you have never seen `eat your broccoli`, estimate

$$\mathtt{P}(\mathtt{broccoli}|\mathtt{eat\ your}) = \mathtt{P}(\mathtt{FOOD}|\mathtt{eat\ your}) \times \mathtt{P}(\mathtt{broccoli}|\mathtt{FOOD})$$

- Clustering can be

hard
: a word belongs to only one cluster (simplifies the model)

soft
: words can be assigned to clusters probabilistically (more flexible)

Skipping

- The contexts
  - `boring|the lecture was`
  - `boring|(the) lecture yesterday was`

  are completely different for an n-gram model
- A potential solution is to consider contexts with gaps, 'skipping' one or more words
- We would, for example model $P(e|abcd)$ with a combination (e.g., interpolation) of
  - $P(e|abc\_)$
  - $P(e|ab\_d)$
  - $P(e|a\_cd)$
  - …

Modeling sentence types

- Another way to improve a language model is to condition on the sentence types
- The idea is different types of sentences (e.g., ones related to different topics) have different behavior
- Sentence types are typically based on clustering
- We create multiple language models, one for each sentence type
- Often a 'general' language model is used, as a fall-back

Caching

- If a word is used in a document, its probability of being used again is high
- Caching models condition the probability of a word, to a larger context (besides the immediate history), such as
  - the words in the document (if document boundaries are marked)
  - a fixed window around the word

Structured langauge models

- Another possibility is usign a generative parser
- Parsers try to explicitly model (good) sentences
- Parser naturally capture long-distance depencencies
- Parsers require much more computational resources than the n-gram models
- The improvements are often small (if any)

Maximum entropy models

- We can fit a logistic regression 'max-ent' model predicting $P(w|\text{context})$
- Main advantage is to be able to condition on arbitrary features

Neural language models

- A neural network can be trained to predict a word from its context
- Then we can use the network for estimating the $P(w|\text{context})$
- In the process, the hidden layer(s) of a network will learn internal representations for the word
- These representations, known as *embeddings*, are continuous representations that place similar words in the same neighborhood in a high-dimensional space
- We will return to embeddings later in this course

Some notes on implementation

- The typical use of n-gram models are on (very) large corpora
- We often need care for numeric instability issues:
  - For example, often it is more convenient to work with 'log probabilities'
  - Sometimes (log) probabilities 'binned' into integers with small number of bits,
- Memory or storage may become a problem too
  - Assuming words below a frequency are 'unknown' often helps
  - Choice of correct data structure becomes important,
  - A common data structure is a *trie* or a *suffix tree*

N-grams, so far ...

- N-gram language models are one of the basic tools in NLP
- They capture some linguistic (and non-linguistic) regularities that are useful in many applications
- The idea is to estimate the probability of a sentence based on its parts (sequences of *words*)
- N-grams are n consecutive units in a sequence
- Typically, we use sequences of *words* to estimate sentence probabilities, but other units are also possible: *characters*, *phonemes*, *phrases*, ...
- For most applications, we introduce sentence boundary markers

N-grams, so far …

- The most straightforward method for estimating probabilities is using relative frequencies (leads to MLE)
- Due to Zipf's law, as we increase 'n', the counts become smaller (data sparseness), many counts become 0
- If there are unknown words, we get 0 probabilities for both words and sentences
- In practice, bigrams or trigrams are used most commonly, applications/datasets of up to 5-grams are also used

N-grams, so far …

- Two different ways of evaluating n-gram models:

Extrinsic success in an external application

Intrinsic likelihood, (cross) entropy, perplexity

- Intrinsic evaluation metrics often correlate well with the extrinsic metrics
- Test your n-grams models on an 'unseen' test set

N-grams, so far …

- Smoothing methods solve the zero-count problem (also reduce the variance)
- Smoothing takes away some probability mass from the observed n-grams, and assigns it to unobserved ones
  - Additive smoothing: add a constant α to all counts
    - $\alpha = 1$ (Laplace smoothing) simply adds one to all counts – simple but often not very useful
    - A simple correction is to add a smaller α, which requires tuning over a development set
  - Discounting removes a fixed amount of probability mass, ϵ, from the observed n-grams
    - We need to re-normalize the probability estimates
    - Again, we need a development set to tune ϵ
  - Good-Turing discounting reserves the probability mass to the unobserved events based on the n-grams seen only once: $p_0 = \frac{n_1}{n}$

N-grams, so far …

- Interpolation and back-off are methods that make use of lower order n-grams in estimating probabilities of higher order n-grams
- In back-off, we fall back to the lower order n-gram if higher order n-gram has 0 counts
- In interpolation, we always use a linear combination of all available n-grams
- We need to adjust higher order n-gram probabilities, to make sure the probabilities sum to one
- A common practice is to use word- or context-sensitive hyperparameters

N-grams, so far … (cont.)

- Two popular methods:
  - Katz back-off uses Good-Turing discounting to reserve the probability mass for lower order n-grams
  - Kneser-Ney interpolation uses absolute discounting, and estimates the lower order / 'back-off' probabilities based on the number of different contexts the word appears
- Normally, the same ideas are applicable for both interpolation and back-off
- There are many other smoothing/interpolation/back-off methods

N-grams, so far … (cont.)

- There are also a few other approaches to language modeling:
  - Skipping models condition the probability words on contexts where some words removed from the context
  - Clustering makes use of probability of 'class' of the word for estimating its probability
  - Sentence types/classes/clusters are also useful in n-gram language modeling
  - Maximum-entropy models (multi-class logistic regression) is another possibility for estimating the probability of a word conditioned on many other features (including the context)
  - Maximum-entropy models (multi-class logistic regression)
  - Neural language models are another approach where the model learns continuous vector representations

Additional reading, references, credits

- Textbook reference: Jurafsky and Martin (2009, chapter 4) (draft chapter for the 3rd version is also available)
- Chen and J. Goodman (1998) and Chen and J. Goodman (1999) include a detailed comparison of smoothing methods. The former (technical report) also includes a tutorial introduction
- J. T. Goodman (2001) studies a number of improvements to (n-gram) language models we have discussed. This technical report also includes some introductory material
- Gale and Sampson (1995) introduce the 'simple' Good-Turing estimation noted on Slide 19. The article also includes an introduction to the basic method.

Additional reading, references, credits (cont.)

- The quote from *2001: A Space Odyssey*, 'I'm sorry Dave. I'm afraid I can't do it.' is probably one of the most frequent quotes in the CL literature. It was also quoted, among many others, by Jurafsky and Martin (2009).
- The HAL9000 camera image on page 19 is from Wikipedia, (re)drawn by Wikipedia user Cryteria.
- The Herman comic used in slide 4 is also a popular example in quite a few lecture slides posted online, it is difficult to find out who was the first.

Chen, Stanley F and Joshua Goodman (1998). *An empirical study of smoothing techniques for language modeling*. Tech. rep. TR-10-98. Harvard University, Computer Science Group. URL: https://dash.harvard.edu/handle/1/25104739.

— (1999). "An empirical study of smoothing techniques for language modeling". In: *Computer speech & language* 13.4, pp. 359–394.

Chomsky, Noam (1968). "Quine's empirical assumptions". In: *Synthese* 19.1, pp. 53–68. DOI: 10.1007/BF00568049.

Additional reading, references, credits (cont.)

Gale, William A and Geoffrey Sampson (1995). "Good-Turing frequency estimation without tears". In: *Journal of Quantitative Linguistics* 2.3, pp. 217–237.

Goodman, Joshua T (2001). *A bit of progress in language modeling extended version*. Tech. rep. MSR-TR-2001-72. Microsoft Research.

Jurafsky, Daniel and James H. Martin (2009). *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition*. second. Pearson Prentice Hall. ISBN: 978-0-13-504196-3.

Shillcock, Richard (1995). "Lexical Hypotheses in Continuous Speech". In: *Cognitive Models of Speech Processing*. Ed. by Gerry T. M. Altmann. MIT Press.