

# Statistical Natural Language Processing

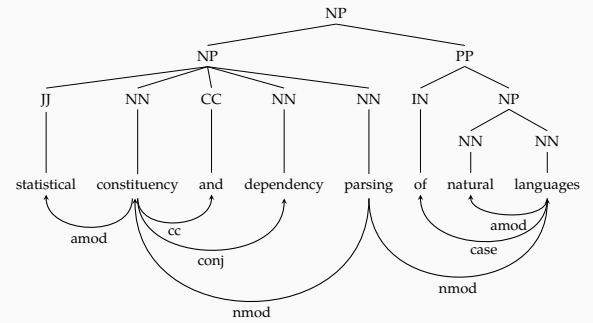
## Statistical Parsing

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Summer Semester 2017

## Next few lectures are about



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## Why do we need syntactic parsing?

- Syntactic analysis is an intermediate step in (semantic) interpretation of sentences



As result, it is useful for applications like *question answering, information extraction, ...*

- (Statistical) parsers are also used as *language models* for applications like *speech recognition* and *machine translation*
- It can be used for *grammar checking*, and can be a useful tool for linguistic research

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## Ingredients of a parser

- A **grammar**
- An algorithm for parsing
- A method for ambiguity resolution

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## Formal grammars

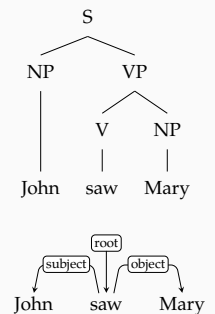
- A formal grammar is a finite specification of a (possibly infinite) language
- We are interested in two broad classes of grammars  
Constituency (or phrase structure) grammars  
Dependency grammars
- Various theories of 'grammar' (e.g., HPSG, LFG, CCG) use ideas/notions from both
- We will study these grammars in their relation to parsing, we do not study or focus on any specific theory

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## Dependency vs. constituency

- Constituency grammars are based on units formed by a group of lexical items (constituents or phrases)
- Dependency grammars model binary head–dependent relations between words
- Most of the theory of parsing is developed with constituency grammars
- Dependency grammars has recently become popular in CL



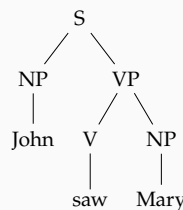
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## Constituency grammars

- Constituency grammars are probably the most studied grammars both in linguistics, and computer science
- The main idea is that a group of words form natural groups, or 'constituents', like *noun phrases* or *word phrases*
- *phrase structure grammars* or *context-free grammars* are often used as synonyms

Note: many grammar formalisms use constituency grammars in some way, we will not focus on a particular grammar formalism here.



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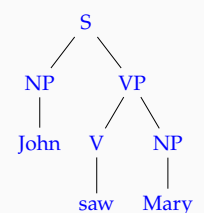
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## Formal definition

A phrase structure grammar is a tuple  $(\Sigma, N, S, R)$

- $\Sigma$  is a set of terminal symbols
- $N$  is a set of non-terminal symbols
- $S \in N$  is a distinguished *start* symbol
- $R$  is a set of rules of the form  $\alpha A \beta \rightarrow \gamma$  for  $A \in N$ ,  $\alpha, \beta, \gamma \in \Sigma \cup N$

- The grammar accepts a sentence if it can be derived from  $S$  with the rewrite rules  $R$

$$\begin{array}{lcl} S & \rightarrow & NP VP \\ NP & \rightarrow & John \mid Mary \\ VP & \rightarrow & V NP \\ V & \rightarrow & saw \end{array}$$


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## Example derivation

The example grammar:

$$\begin{array}{l} S \rightarrow NP VP \quad VP \rightarrow V NP \\ NP \rightarrow \text{John} \mid \text{Mary} \quad V \rightarrow \text{saw} \end{array}$$

- Phrase structure grammars derive a sentence with successive application of rewrite rules.  
 $S \Rightarrow NP VP \Rightarrow \text{John VP} \Rightarrow \text{John V NP} \Rightarrow \text{John saw NP} \Rightarrow \text{John saw Mary}$   
or,  $S \Rightarrow \text{John saw Mary}$
- The intermediate forms that contain non-terminals are called *sentential forms*

## Chomsky hierarchy of grammars

- type 0 Recursively enumerable, recognized by Turing machines (HPSG, LFG)  
 $\alpha A \beta \rightarrow \gamma$
- type 1 Context sensitive, recognized by linear-bound automaton  
 $\alpha A \beta \rightarrow \alpha \gamma \beta, \quad \gamma \neq \epsilon$
- type 2.1 Mildly context sensitive (TAG, CCG)
- type 2 Context free, recognized by push-down automata  
 $A \rightarrow \alpha$
- type 3 Regular, recognized by finite-state automata  
 $A \rightarrow aB \quad \text{or} \quad A \rightarrow Ba$

In all of the above  $A$  and  $B$  are non-terminals,  $a$  is a terminal symbol,  $\alpha, \beta, \gamma$  are sequences of terminals and non-terminals, and  $\epsilon$  is the empty string.

## Some examples

- Regular grammars (finite-state automata) do not have any memory
  - can represent  $a^*b^*$ , but not  $a^n b^n$
- Finite-state automata are used in many tasks in CL, including morphological analysis, partial parsing
- Context free grammars (push-down automata) use a stack
  - can represent  $a^n b^n, a^n b^m c^m d^n$ , but not  $a^n b^m c^n d^m$
- Context-free grammars form the basis of most parsers
- Context-sensitive languages can do all of the above, but they are too powerful, and computationally expensive

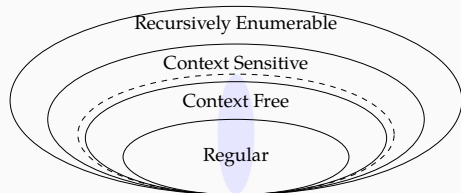
## Expressiveness of grammar classes

- The class of grammars adequate for formally describing natural languages has been an important question for (computational) linguistics
- For the most part, context-free grammars are enough, but there are some examples, e.g., from Swiss German (Shieber 1985) Jan säit das...

...mer **em** Hans **es** huss **h**älfed **a**aasriiche  
...we Hans (DAT) house (ACC) helped paint

Note that this resembles  $a^n b^m c^n d^m$ .

## Chomsky hierarchy: the picture



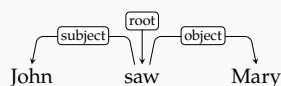
- Chomsky hierarchy of languages form a hierarchy (with some care about empty language)
- It is often claimed that mildly context sensitive grammars (dashed ellipse) are adequate for representing natural languages
- Note, however, not even every regular language is a potential natural language (e.g.,  $a^*bbc^*$ ). The possible natural languages probably cross-cut this hierarchy (shaded region)

## Constituency grammars and parsing

- Context-free grammars are parseable in  $O(n^3)$  time complexity using dynamic programming algorithms
- Mildly context-sensitive grammars can also be parsed in polynomial time ( $O(n^6)$ )
- Polynomial time algorithms are not always good enough in practice
  - We often use approximate solutions with greedy search algorithms

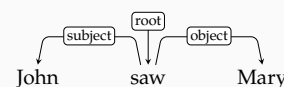
## Dependency grammars

- Dependency grammars gained popularity in (particularly in computational) linguistics rather recently, but their roots can be traced back to a few thousand years (modern dependency grammars are attributed to Tesnière 1959)
- The main idea is capturing the relation between the words, rather than grouping them into (abstract) constituents



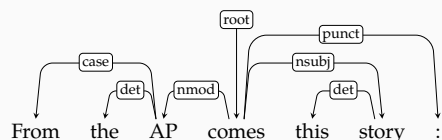
Note: like constituency grammars, we will not focus on a particular dependency formalism, but discuss it in general in relation to parsing.

## Properties of dependency grammars



- The structure of the sentence is represented by asymmetric binary links between *lexical items*
- Each relation defines one of the words as the *head* and the other as *dependent*
- The links (relations) have labels (dependency types)
- Most dependency grammar require each word to have only a single head

## A more realistic example



## How to determine heads

1. *Head* (H) determines the syntactic category of the *construction* (C) and can often replace C
2. H determines the semantic category of C; the *dependent* (D) gives semantic specification
3. H is obligatory, D may be optional
4. H selects D and determines whether D is obligatory or optional
5. The form and/or position of dependent is determined by the head
6. The form of D depends on H
7. The linear position of D is specified with reference to H

(from Kübler, McDonald, and Nivre 2009, p.3-4)

## Issues with head assignment and dependency labels

- Like the tests for constituency, determining heads are not always straightforward
- A construction is called *endocentric* if the head can replace the whole construction, *exocentric* otherwise



- It is often unclear whether dependency labels encode syntactic or semantic functions

## Some tricky constructions

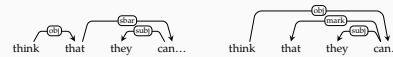
- Coordination



- Prepositional phrases



- Subordinate clauses



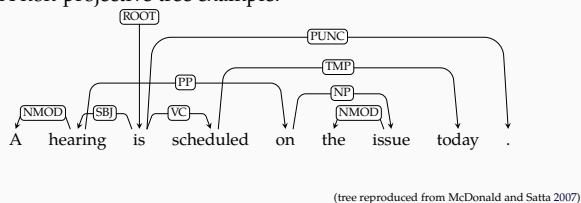
- Auxiliaries vs. main verbs



## Projective vs. non-projective dependencies

- If a dependency graph has no crossing edges, it is said to be *projective*, otherwise *non-projective*
- Non-projectivity stems from long-distance dependencies and free word order

A non-projective tree example:



## Parsing with dependency grammars

- Projective dependency parsing can be done in polynomial time
- Non-projective parsing is NP-hard (without restrictions)
- For both, it is a common practice to use greedy (e.g., linear time) algorithms

## Dependency vs. constituency

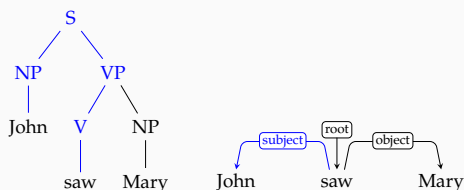
- Constituency grammars are based on units formed by a group of lexical items (constituents or phrases)
- Dependency grammars model binary head-dependent relations between words
- Most of the theory of parsing is developed with constituency grammars
- Dependency grammars has recently become more popular in CL
- Note that many formalisms and treebanks follow a hybrid approach, using ideas from both

## Where do grammars come from

- Grammars for (statistical) parsing can be either
  - hand crafted (many years of expert effort)
  - extracted from *treebanks* (which also require lots of effort)
  - 'induced' from raw data (interesting, but not as successful)
- Current practice relies mostly on treebanks
- Hybrid approaches also exist
- Grammar induction is not common (for practical models) but exploiting unlabelled data is also a common trend

## Conversion between constituencies and dependencies

- Although non-trivial, conversion between dependency and constituency annotation is possible
- One can take the path between two words as a dependency relation



- The conversion from constituencies to dependencies is a common practice in the field

## Grammars for natural language parsing: summary

- A grammar is a formal device for specifying a language
- Grammars are one of the important components of a parser, they can be hand-crafted or extracted from a treebank
- Most of the parsing theory and practice is based on constituency, particularly context-free grammars
- Dependency grammars have become more popular recently, and often easier to use in NLP applications

## Context free grammars

recap

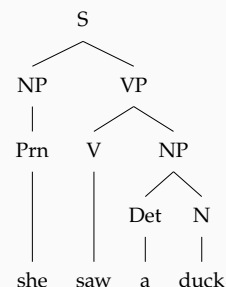
- Context free grammars are sufficient for expressing most phenomena in natural language syntax
- Most of the parsing theory (and practice) is built on parsing CF languages
- The context-free rules have the form

$$A \rightarrow \alpha$$

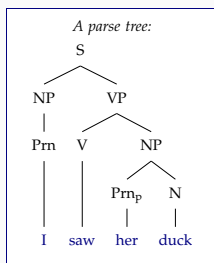
where  $A$  is a single non-terminal symbol and  $\alpha$  is a (possibly empty) sequence of terminal or non-terminal symbols

## An example context-free grammar

$S \rightarrow NP VP$	Derivation of sentence 'she saw a duck'
$S \rightarrow Aux NP VP$	$S \Rightarrow NP VP$
$NP \rightarrow Det N$	$NP \Rightarrow Prn$
$NP \rightarrow Prn$	$Prn \Rightarrow she$
$NP \rightarrow NP PP$	$VP \Rightarrow V NP$
$VP \rightarrow V NP$	$V \Rightarrow saw$
$VP \rightarrow V$	$NP \Rightarrow Det N$
$VP \rightarrow VP PP$	$Det \Rightarrow a$
$PP \rightarrow Prp NP$	$N \Rightarrow duck$
$N \rightarrow duck$	
$N \rightarrow park$	
$N \rightarrow parks$	
$V \rightarrow duck$	
$V \rightarrow ducks$	
$V \rightarrow saw$	
$Prn \rightarrow she \mid her$	
$Prp \rightarrow in \mid with$	
$Det \rightarrow a \mid the$	



## Representations of a context-free parse tree



A history of derivations:

- $S \Rightarrow NP VP$
- $NP \Rightarrow Prn$
- $Prn \Rightarrow I$
- $VP \Rightarrow V NP$
- $V \Rightarrow saw$
- $NP \Rightarrow Prn_p N$
- $Prn_p \Rightarrow her$
- $N \Rightarrow duck$

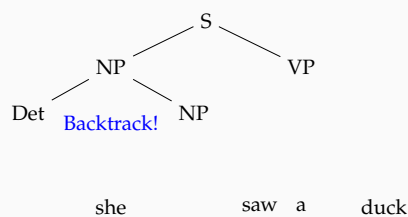
A sequence with (labeled) brackets

$$\left[ S \left[ NP \left[ Prn I \right] \right] \left[ VP \left[ V saw \right] \left[ NP \left[ Prn_p her \right] \left[ N duck \right] \right] \right] \right]$$

## Parsing as search

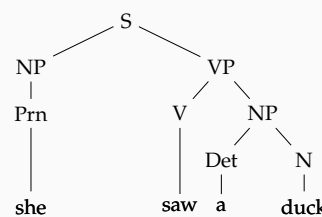
- Parsing can be seen as search constrained by the grammar and the input
- Top down: start from  $S$ , find the derivations that lead to the sentence
- Bottom up: start from the sentence, find series of derivations (in reverse) that leads to  $S$
- Search can be depth first or breadth first for both cases

## Parsing as search: top down



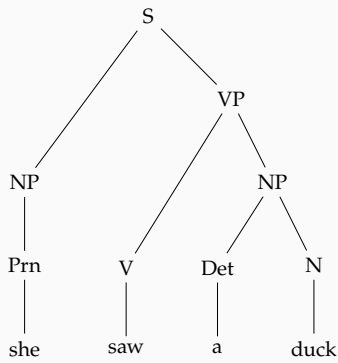
$S \rightarrow NP VP$
$S \rightarrow Aux NP VP$
$NP \rightarrow Det N$
$NP \rightarrow Prn$
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$VP \rightarrow V$
$VP \rightarrow VP PP$
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## Parsing as search: top down



$S \rightarrow NP VP$
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$N \rightarrow parks$
$V \rightarrow duck$
$V \rightarrow ducks$
$V \rightarrow saw$
$Prn \rightarrow she \mid her$
$Prp \rightarrow in \mid with$
$Det \rightarrow a \mid the$

## Parsing as search: bottom up



- S → NP VP
- S → Aux NP VP
- NP → Det N
- NP → Prn
- NP → NP PP
- VP → V NP
- VP → V
- VP → VP PP
- PP → Prp NP
- N → duck
- N → park
- N → parks
- V → duck
- V → ducks
- V → saw
- Prn → she | her
- Prp → in | with
- Det → a | the

## Problems with search procedures

- Top-down search considers productions incompatible with the input, and cannot handle left recursion
  - Bottom-up search considers non-terminals that would never lead to S
  - Repeated work because of backtracking
- The result is exponential time complexity in the length of the sentence

Some of these problems can be solved using *dynamic programming*.

## CKY algorithm

- The CKY (Cocke–Younger–Kasami), or CYK, parsing algorithm is a dynamic programming algorithm (Kasami 1965; Younger 1967; Cocke and Schwartz 1970)
- It processes the input *bottom up*, and saves the intermediate results on a *chart*
- Time complexity for *recognition* is  $O(n^3)$  (with a space complexity of  $O(n^2)$ )
- It requires the CFG to be in *Chomsky normal form* (CNF)

## Chomsky normal form (CNF)

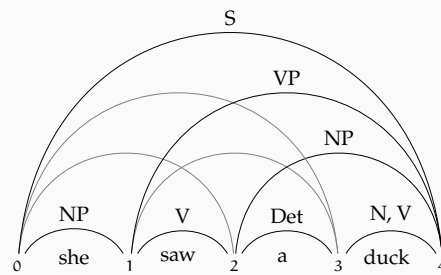
- A CFG is in CNF, if the rewrite rules are in one of the following forms
  - $A \rightarrow BC$
  - $A \rightarrow a$
 where A, B, C are non-terminals and a is a terminal
- Any CFG can be converted to CNF
- Resulting grammar is *weakly equivalent* to the original grammar:
  - it generates/accepts the same language
  - but the derivations are different

## Converting to CNF: example

- For rules with > 2 RHS symbols  
 $S \rightarrow \text{Aux NP VP} \Rightarrow \bar{S} \rightarrow \text{Aux } X$   
 $X \rightarrow \text{NP VP}$
- For rules with < 2 RHS symbols  
 $\text{NP} \rightarrow \text{Prn} \Rightarrow \text{NP} \rightarrow \text{she} \mid \text{her}$

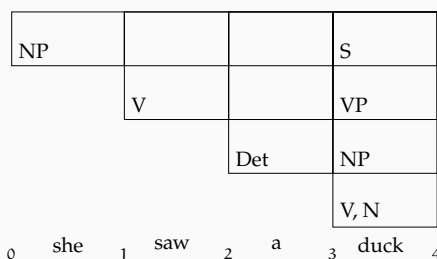
- S → NP VP
- S → Aux NP VP
- NP → Det N
- NP → Prn
- NP → NP PP
- VP → V NP
- VP → V
- VP → VP PP
- PP → Prp NP
- N → duck
- N → park
- N → parks
- V → duck
- V → ducks
- V → saw
- Prn → she | her
- Prp → in | with
- Det → a | the

## CKY demonstration: spans



- S → NP VP
- S → Aux NP VP
- NP → Det N
- NP → Prn
- NP → NP PP
- VP → V NP
- VP → V
- VP → VP PP
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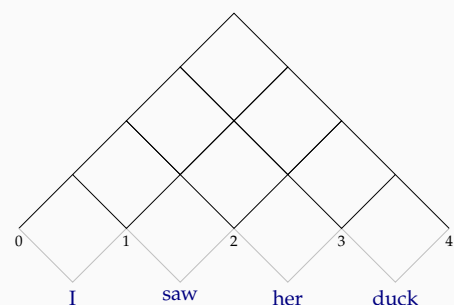
## CKY demonstration: the chart



- S → NP VP
- S → Aux NP VP
- NP → Det N
- NP → Prn
- NP → NP PP
- VP → V NP
- VP → V
- VP → VP PP
- PP → Prp NP
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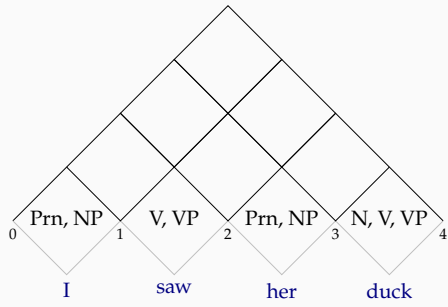
## CKY demonstration

an ambiguous example



### CKY demonstration

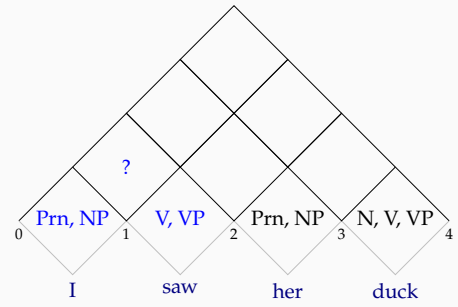
an ambiguous example



### CKY demonstration

an ambiguous example

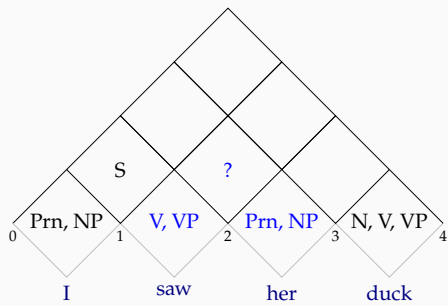
$S \rightarrow NP VP$



### CKY demonstration

an ambiguous example

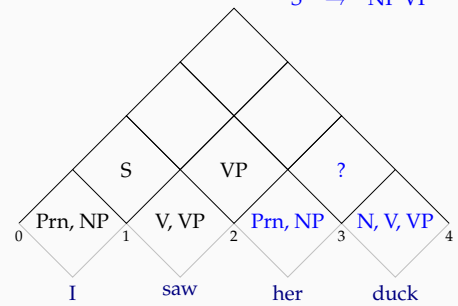
$VP \rightarrow V NP$



### CKY demonstration

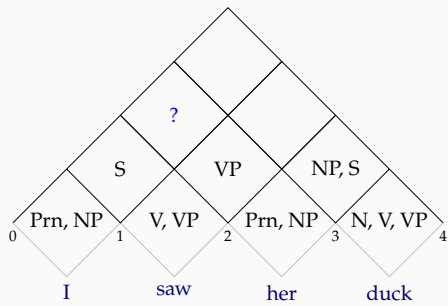
an ambiguous example

$NP \rightarrow Prn N$   
 $S \rightarrow NP VP$



### CKY demonstration

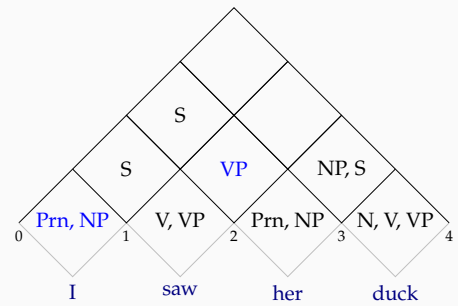
an ambiguous example



### CKY demonstration

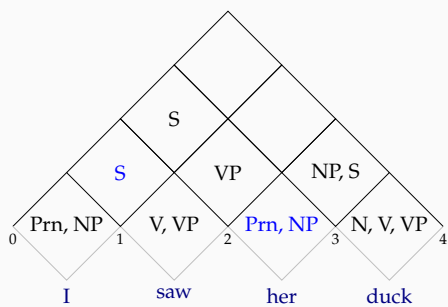
an ambiguous example

$S \rightarrow NP VP$



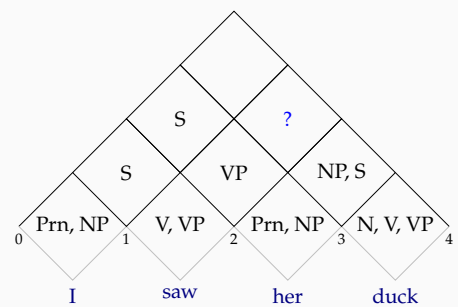
### CKY demonstration

an ambiguous example



### CKY demonstration

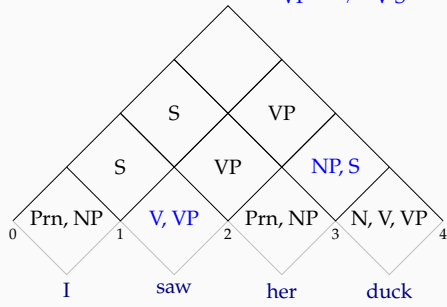
an ambiguous example



### CKY demonstration

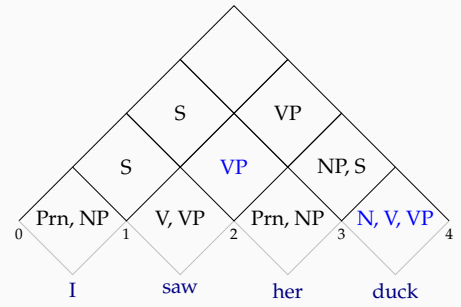
an ambiguous example

VP → V NP  
VP → V S



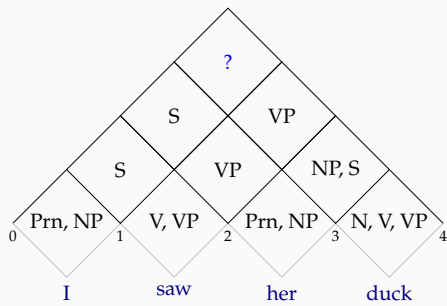
### CKY demonstration

an ambiguous example



### CKY demonstration

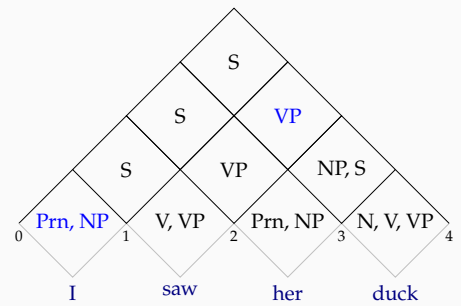
an ambiguous example



### CKY demonstration

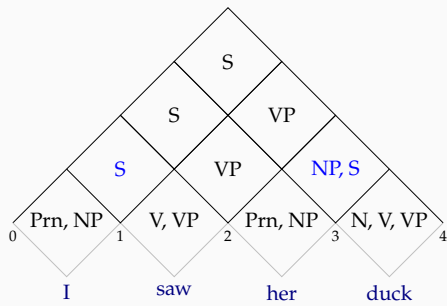
an ambiguous example

S → NP VP



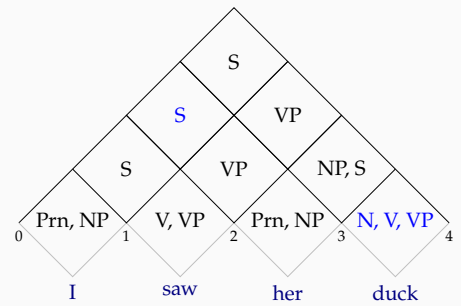
### CKY demonstration

an ambiguous example



### CKY demonstration

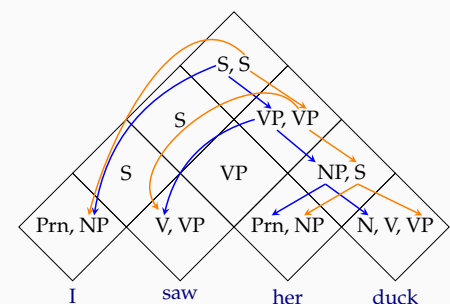
an ambiguous example



### Parsing vs. recognition

- We went through a recognition example
- Recognition accepts or rejects as sentence based on a grammar
- For parsing, we want to know the derivations that yielded a correct parse
- To recover parse trees, we
  - we follow the same procedure as recognition
  - add back links to keep track of the derivations

### Chart parsing example (CKY parsing)



## CKY summary

- + CKY avoids re-computing the analyses by storing analyses of sub-spans in a table
- It still computes lower level constituents that are not allowed by the grammar
- CKY requires the grammar to be in CNF
  - CKY has  $O(n^3)$  recognition complexity
  - For parsing we need to keep track of backlinks
  - CKY can efficiently store all possible parses in a chart
  - Enumerating all possible parses have exponential complexity (worst case)

## Earley algorithm

- Earley algorithm is a top down parsing algorithm (Earley 1970)
- It allows arbitrary CFGs
- Keeps record of constituents that are
  - predicted using the grammar (top-down)
  - in-progress with partial evidence
  - completed based on input seen so far
 at every position in the input string
- Time complexity is  $O(n^3)$

## Earley chart entries (states or items)

Earley chart entries are CF rules with a 'dot' on the RHS representing the state of the rule

- $A \rightarrow \bullet \alpha [i, i]$  predicted without any evidence (yet)
- $A \rightarrow \alpha \bullet \beta [i, j]$  partially matched
- $A \rightarrow \alpha \beta \bullet [i, j]$  completed, the non-terminal  $A$  is found in the given span

## Earley algorithm: an informal sketch

1. Start at position 0, predict  $S$
2. Predict all possible states (rules that apply)
3. Read a word
4. Update the table, advance the dot if possible
5. Go to step 2
6. If we have a completed  $S$  production at the end of the input, the input is recognized

## Earley algorithm: three operations

**Predictor** adds all possible rules that are possible at the given state

**Completer** adds states from the earlier chart entries that match the completed state to the chart entry being processed, and advances their dot

**Scanner** adds a completed state to the next chart entry if the current category is a POS tag, and the word matches

## Earley parsing example (chart[0])

0	she	1	saw	2	a	3	duck	4
state	rule	position	operation					
0	$\gamma \rightarrow \bullet S$	[0,0]	initialization					
1	$S \rightarrow \bullet NP VP$	[0,0]	predictor					
2	$S \rightarrow \bullet Aux NP VP$	[0,0]	predictor					
3	$NP \rightarrow \bullet Det N$	[0,0]	predictor					
4	$NP \rightarrow \bullet NP PP$	[0,0]	predictor					
5	$NP \rightarrow \bullet Prn$	[0,0]	predictor					

$S \rightarrow NP VP$   
 $S \rightarrow Aux NP VP$   
 $NP \rightarrow Det N$   
 $NP \rightarrow Prn$   
 $NP \rightarrow NP PP$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Prp NP$   
 $N \rightarrow duck$   
 $N \rightarrow park$   
 $N \rightarrow parks$   
 $V \rightarrow duck$   
 $V \rightarrow ducks$   
 $V \rightarrow saw$   
 $Prn \rightarrow she | her$   
 $Prp \rightarrow in | with$   
 $Det \rightarrow a | the$   
 $Aux \rightarrow does | has$

## Earley parsing example (chart[1])

0	she	1	saw	2	a	3	duck	4
state	rule	position	operation					
6	$Prn \rightarrow she \bullet$	[0,1]	scanner					
7	$NP \rightarrow Prn \bullet$	[0,1]	completer					
8	$S \rightarrow NP \bullet VP$	[0,1]	completer					
9	$NP \rightarrow NP \bullet PP$	[0,1]	completer					
10	$VP \rightarrow \bullet V NP$	[1,1]	predictor					
11	$VP \rightarrow \bullet VP PP$	[1,1]	predictor					
12	$PP \rightarrow \bullet Prp NP$	[1,1]	predictor					

$S \rightarrow NP VP$   
 $S \rightarrow Aux NP VP$   
 $NP \rightarrow Det N$   
 $NP \rightarrow Prn$   
 $NP \rightarrow NP PP$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Prp NP$   
 $N \rightarrow duck$   
 $N \rightarrow park$   
 $N \rightarrow parks$   
 $V \rightarrow duck$   
 $V \rightarrow ducks$   
 $V \rightarrow saw$   
 $Prn \rightarrow she | her$   
 $Prp \rightarrow in | with$   
 $Det \rightarrow a | the$   
 $Aux \rightarrow does | has$

## Earley parsing example (chart[2])

0	she	1	saw	2	a	3	duck	4
state	rule	position	operation					
13	$V \rightarrow saw \bullet$	[1,2]	scanner					
14	$VP \rightarrow V \bullet NP$	[1,2]	completer					
15	$VP \rightarrow V \bullet$	[1,2]	completer					
16	$NP \rightarrow \bullet Det N$	[2,2]	predictor					
17	$NP \rightarrow \bullet NP PP$	[2,2]	predictor					
18	$NP \rightarrow \bullet Prn$	[2,2]	predictor					
19	$S \rightarrow NP VP \bullet$	[0,2]	predictor					

$S \rightarrow NP VP$   
 $S \rightarrow Aux NP VP$   
 $NP \rightarrow Det N$   
 $NP \rightarrow Prn$   
 $NP \rightarrow NP PP$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Prp NP$   
 $N \rightarrow duck$   
 $N \rightarrow park$   
 $N \rightarrow parks$   
 $V \rightarrow duck$   
 $V \rightarrow ducks$   
 $V \rightarrow saw$   
 $Prn \rightarrow she | her$   
 $Prp \rightarrow in | with$   
 $Det \rightarrow a | the$   
 $Aux \rightarrow does | has$



## Earley parsing example (chart[3])

0	she	1	saw	2	a	3	duck	4
state		rule		position		operation		
20	Det	→a •		[2,3]			scanner	
21	NP	→Det •N		[2,3]			completer	

- S → NP VP
- S → Aux NP VP
- NP → Det N
- NP → Prn
- NP → NP PP
- VP → V NP
- VP → V
- VP → VP PP
- PP → Prp NP
- N → duck
- N → park
- N → parks
- V → duck
- V → ducks
- V → saw
- Prn → she | her
- Prp → in | with
- Det → a | the
- Aux → does | has

## Earley parsing example (chart[4])

0	she	1	saw	2	a	3	duck	4
state		rule		position		operation		
22	N	→duck •		[3,4]			scanner	
23	NP	→Det N •		[2,4]			completer	
24	VP	→V NP •		[1,4]			completer	
25	S	→NP VP •		[0,4]			completer	

- S → NP VP
- S → Aux NP VP
- NP → Det N
- NP → Prn
- NP → NP PP
- VP → V NP
- VP → V
- VP → VP PP
- PP → Prp NP
- N → duck
- N → park
- N → parks
- V → duck
- V → ducks
- V → saw
- Prn → she | her
- Prp → in | with
- Det → a | the
- Aux → does | has

## Summary: context-free parsing algorithms

- Naive search for parsing is intractable
- Dynamic programming algorithms allow polynomial time recognition
- Parsing may still be exponential in the worse case
- Ambiguity: CKY or Earley parse tables can represent ambiguity, but cannot say anything about which parse is the best

## Pretty little girl's school (again)



Cartoon Theories of Linguistics, SpecGram Vol CLIII, No 4, 2008. <http://specgram.com/CLIII.4/school1.gif>

## Some more examples

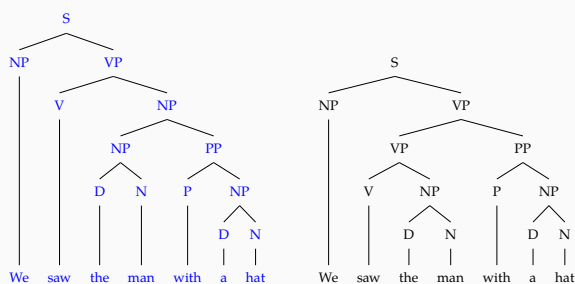
- Lexical ambiguity
  - She is looking for a match
  - We saw her duck
- Attachment ambiguity
  - I saw the man with a telescope
  - Panda eats bamboo shoots and leaves
- Local ambiguity (garden path sentences)
  - The horse raced past the barn fell
  - The old man the boats
  - Fat people eat accumulates
- Anaphora resolution
  - Every farmer who owns a donkey beats it.

## Even more examples

(newspaper headlines)

- FARMER BILL DIES IN HOUSE
- TEACHER STRIKES IDLE KIDS
- SQUAD HELPS DOG BITE VICTIM
- BAN ON NUDE DANCING ON GOVERNOR'S DESK
- PROSTITUTES APPEAL TO POPE
- KIDS MAKE NUTRITIOUS SNACKS
- DRUNK GETS NINE MONTHS IN VIOLIN CASE
- MINERS REFUSE TO WORK AFTER DEATH

## The task: choosing the most plausible parse



## Statistical parsing

- Find the most plausible parse of an input string given all possible parses
- We need a scoring function, for each parse, given the input
- We typically use probabilities for scoring, task becomes finding the parse (or tree),  $t$ , given the input string  $x$

$$t_{\text{best}} = \arg \max_t P(t|x)$$

- Note that some ambiguities need a larger context than the sentence to be resolved correctly

## Probabilistic context free grammars (PCFG)

A probabilistic context free grammar is specified by,

- $\Sigma$  is a set of terminal symbols
- $N$  is a set of non-terminal symbols
- $S \in N$  is a distinguished *start* symbol
- $R$  is a set of rules of the form

$$A \rightarrow \alpha [p]$$

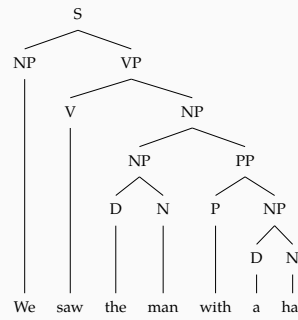
where  $A$  is a non-terminal,  $\alpha$  is string of terminals and non-terminals, and  $p$  is the probability associated with the rule

- The grammar accepts a sentence if it can be derived from  $S$  with rules  $R_1 \dots R_k$
- The probability of a parse  $t$  of input string  $x$ ,  $P(t|x)$ , corresponding to the derivation  $R_1 \dots R_k$  is

$$P(t|x) = \prod_1^k p_i$$

where  $p_i$  is the probability of the rule  $R_i$

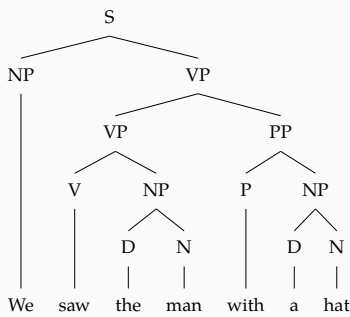
## PCFG example (1)



$S \rightarrow NP VP$	1.0
$NP \rightarrow D N$	0.7
$NP \rightarrow NP PP$	0.2
$NP \rightarrow We$	0.1
$VP \rightarrow V NP$	0.9
$VP \rightarrow VP PP$	0.1
$PP \rightarrow P NP$	1.0
$N \rightarrow hat$	0.2
$N \rightarrow man$	0.8
$V \rightarrow saw$	1.0
$P \rightarrow with$	1.0
$D \rightarrow a$	0.6
$D \rightarrow the$	0.4

$$P(t) = 1.0 \times 0.1 \times 0.9 \times 1.0 \times 0.2 \times 0.7 \times 0.4 \times 0.8 \times 1.0 \times 1.0 \times 0.7 \times 0.6 \times 0.2 = 0.000263424$$

## PCFG example (2)



$S \rightarrow NP VP$	1.0
$NP \rightarrow D N$	0.7
$NP \rightarrow NP PP$	0.2
$NP \rightarrow We$	0.1
$VP \rightarrow V NP$	0.9
$VP \rightarrow VP PP$	0.1
$PP \rightarrow P NP$	1.0
$N \rightarrow hat$	0.2
$N \rightarrow man$	0.8
$V \rightarrow saw$	1.0
$P \rightarrow with$	1.0
$D \rightarrow a$	0.6
$D \rightarrow the$	0.4

$$P(t) = 1.0 \times 0.1 \times 0.3 \times 0.7 \times 1.0 \times 0.1 \times 0.8 \times 0.4 \times 0.8 \times 1.0 \times 1.0 \times 0.7 \times 0.6 \times 0.2 = 0.0001317120$$

## Where do the rule probabilities come from?

- Supervised: estimate from a treebank, e.g., using maximum likelihood estimation
- Unsupervised: expectation-maximization (EM)

## PCFGs - an interim summary

- PCFGs assign probabilities to parses based on CFG rules used during the parse
- PCFGs assume that the rules are independent
- PCFGs are generative models, they assign probabilities to  $P(t, x)$ , we can calculate the probability of a sentence by

$$P(x) = \sum_t P(t, x) = \sum_t P(t)$$

## What makes the difference in PCFG probabilities?

$S \Rightarrow NP VP$	1.0	$S \Rightarrow NP VP$	1.0
$NP \Rightarrow We$	0.1	$NP \Rightarrow We$	0.1
$VP \Rightarrow VP PP$	0.1	$VP \Rightarrow V NP$	0.7
$VP \Rightarrow V NP$	0.8	$V \Rightarrow saw$	1.0
$V \Rightarrow saw$	1.0	$NP \Rightarrow NP PP$	0.2
$NP \Rightarrow D N$	0.7	$NP \Rightarrow D N$	0.7
$D \Rightarrow the$	0.4	$D \Rightarrow the$	0.4
$N \Rightarrow man$	0.8	$N \Rightarrow man$	0.8
$PP \Rightarrow P NP$	1.0	$PP \Rightarrow P NP$	1.0
$P \Rightarrow with$	1.0	$P \Rightarrow with$	1.0
$NP \Rightarrow D N$	0.7	$NP \Rightarrow D N$	0.7
$D \Rightarrow a$	0.6	$D \Rightarrow a$	0.6
$N \Rightarrow hat$	0.2	$N \Rightarrow hat$	0.2

The parser's choice would not be affected by lexical items!

## What is wrong with PCFGs?

- In general: the assumption of independence
- The parents affect the correct choice for children, for example, in English  $NP \rightarrow Prn$  is more likely in the subject position
- The lexical units affect the correct choice decision, for example:
  - We eat the pizza with hands
  - We eat the pizza with mushrooms
- Additionally: PCFGs use local context, difficult to incorporate arbitrary/global features for disambiguation

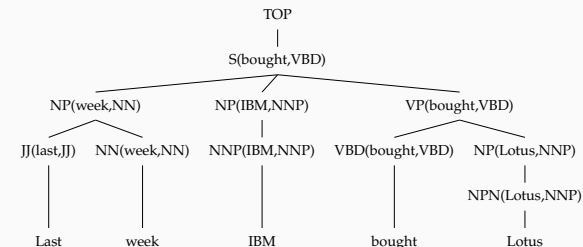
## Solutions to PCFG problems

- Independence assumptions can be relaxed by either
  - Parent annotation
  - Lexicalization - Collins (1999)
- To condition on arbitrary/global information: discriminative models - Charniak and Johnson (2005)
- Most practical PCFG parsers are lexicalized, and often use a re-ranker conditioning on other (global) features

## Lexicalizing PCFGs

- Replace non-terminal  $X$  with  $X(h)$ , where  $h$  is a tuple with the lexical word and its POS tag
- Now the grammar can capture (head-driven) lexical dependencies
- But number of nonterminals grow by  $|V| \times |T|$
- Estimation becomes difficult (many rules, data sparsity)
- Some treebanks (e.g., Penn Treebank) do not annotate heads, they are automatically annotated (based on heuristics)

## Example lexicalized derivation



### Example rules:

TOP  $\rightarrow$  S(bought,VBD)  
 S(bought,VBD)  $\rightarrow$  NP(week,NN) NP(IBM,NNP) VP(bought,VBD)  
 VP(bought,VBD)  $\rightarrow$  VBD(bought,VBD) NP(Lotus,NNP)  
 JJ(last,JJ)  $\rightarrow$  Last

## Evaluating the parser output

- A parser can be evaluated
  - extrinsically based on its effect on a task (e.g., machine translation) where it is used
  - intrinsically based on the match with ideal parsing
- The typically evaluation (intrinsic) is based on a *gold standard* (GS)
- Exact match is often
  - very difficult to achieve (think about a 50-word newspaper sentence)
  - not strictly necessary (recovering parts of the parse can be useful for many purposes)

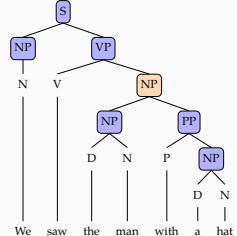
## Parser evaluation metrics

- Common evaluation metrics are (PARSEVAL):
  - precision the ratio of correctly predicted nodes
  - recall the nodes (in GS) that are predicted correctly
  - f-measure harmonic mean of precision and recall
 
$$\left( \frac{2 \times \text{precision} \times \text{recall}}{\text{precision} + \text{recall}} \right)$$
- The measures can be
  - unlabeled the spans of the nodes are expected to match
  - labeled the node label should also match
- Crossing brackets (or average non-crossing brackets)
 

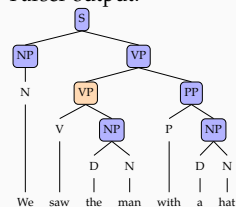
( We ( saw ( them ( with binoculars )))  
 ( We (( saw them ) ( with binoculars )))
- Measures can be averaged per constituent (micro average), or over sentences (macro average)

## PARSEVAL example

### Gold standard:



### Parser output:



$$\text{precision} = \frac{6}{7} \quad \text{recall} = \frac{6}{7} \quad \text{f-measure} = \frac{6}{7}$$

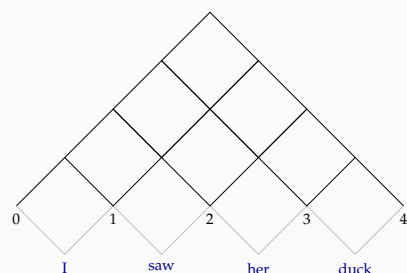
## Problems with PARSEVAL metrics

- PARSEVAL metrics favor certain type of structures
  - Results are surprisingly well for flat tree structures (e.g., Penn treebank)
  - Results of some mistakes are catastrophic (e.g., low attachment)
- Not all mistakes are equally important for semantic distinctions
- Some alternatives:
  - Extrinsic evaluation
  - Evaluation based on extracted dependencies

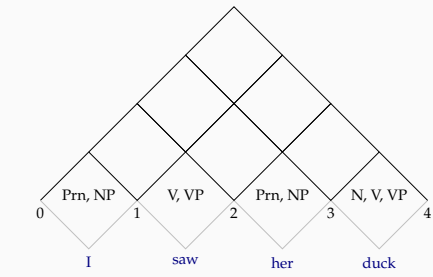
## PCFG chart parsing

- Both CKY and Earley algorithms can be adapted to PCFG parsing
- CKY matches PCFG parsing quite well
  - to get the best parse, store the constituent with the highest probability in every cell the chart
  - to get n-best best parse (beam search), store the n-best constituents in every cell in the chart

## CKY for PCFG parsing



### CKY for PCFG parsing



$$P(\text{Prn}_{01}) = P(\text{Prn} \rightarrow \text{I})$$

$$P(\text{NP}_{01}) = P(\text{NP} \rightarrow \text{I})$$

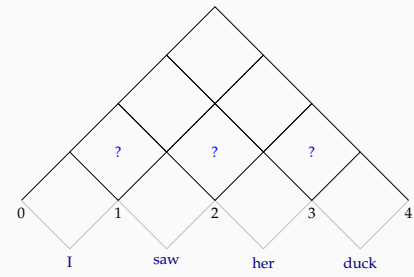
$$P(\text{V}_{12}) = P(\text{V} \rightarrow \text{saw})$$

$$P(\text{VP}_{12}) = P(\text{VP} \rightarrow \text{saw})$$

...

### CKY for PCFG parsing

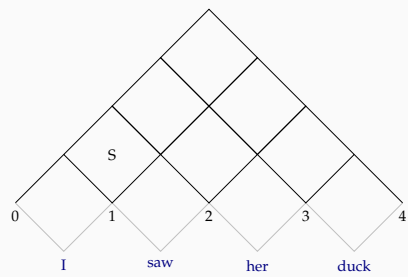
$$S \rightarrow \text{NP VP}$$



$$P(S_{02} \Rightarrow \text{NP}_{01} \text{VP}_{12}) = P(\text{NP}_{01})P(\text{VP}_{12})P(S \rightarrow \text{NP VP})$$

### CKY for PCFG parsing

$$\text{VP} \rightarrow \text{V NP}$$

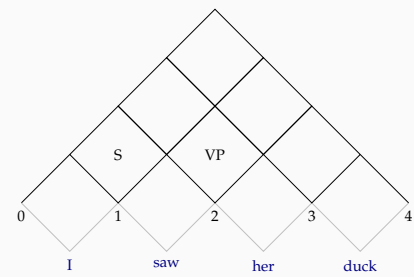


$$P(\text{VP}_{13} \Rightarrow \text{V}_{12} \text{NP}_{23}) = P(\text{V}_{12})P(\text{NP}_{23})P(\text{VP} \rightarrow \text{V NP})$$

### CKY for PCFG parsing

$$\text{NP} \rightarrow \text{Prn N}$$

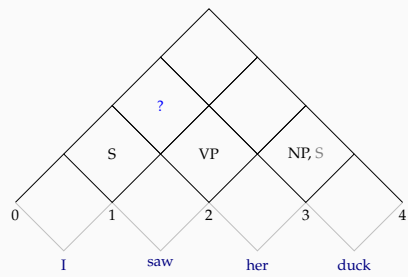
$$S \rightarrow \text{NP VP}$$



$$P(\text{NP}_{24} \Rightarrow \text{Prn}_{23} \text{N}_{34}) = P(\text{Prn}_{23})P(\text{N}_{34})P(\text{Prn} \rightarrow \text{Prn N})$$

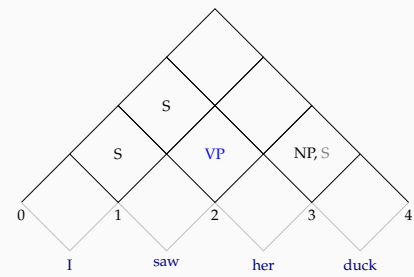
$$P(S_{24} \Rightarrow \text{NP}_{23} \text{VP}_{34}) = P(\text{NP}_{23})P(\text{VP}_{34})P(S \rightarrow \text{NP VP})$$

### CKY for PCFG parsing



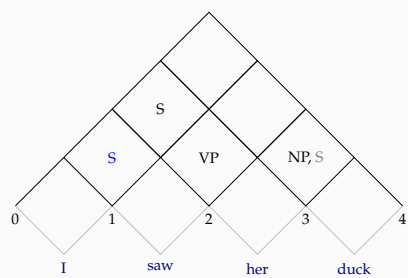
### CKY for PCFG parsing

$$S \rightarrow \text{NP VP}$$

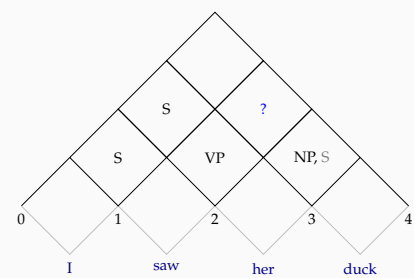


$$P(S_{03} \Rightarrow \text{NP}_{01} \text{VP}_{13}) = P(\text{NP}_{01})P(\text{VP}_{13})P(S \rightarrow \text{NP VP})$$

### CKY for PCFG parsing

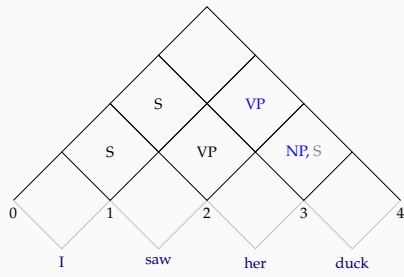


### CKY for PCFG parsing



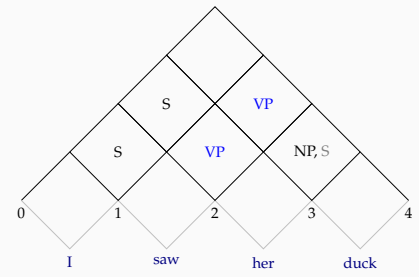
### CKY for PCFG parsing

VP → V NP  
 VP → V S

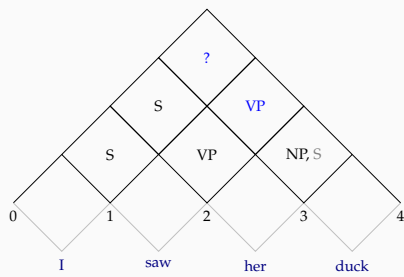


$$P(VP_{14} \Rightarrow V_{12}NP_{24}) = P(V_{12})P(NP_{24})P(VP \rightarrow V NP)$$

### CKY for PCFG parsing

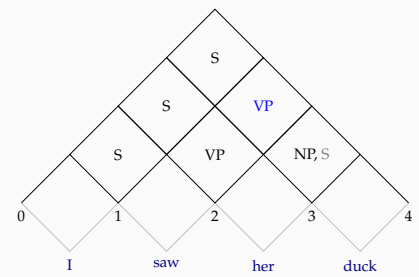


### CKY for PCFG parsing



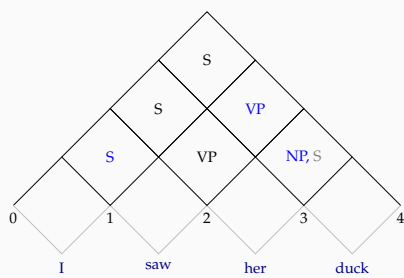
### CKY for PCFG parsing

S → NP VP

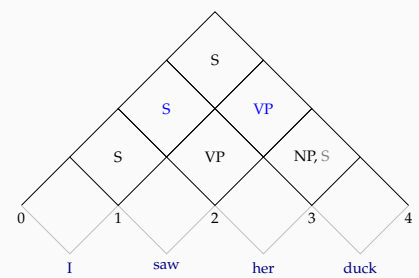


$$P(S_{14} \Rightarrow NP_{01}VP_{14}) = P(NP_{01})P(VP_{14})P(S \rightarrow NP VP)$$

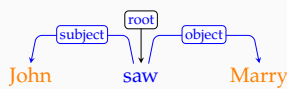
### CKY for PCFG parsing



### CKY for PCFG parsing

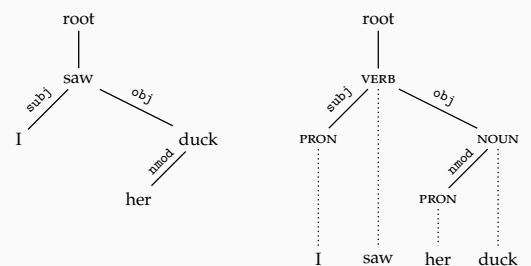


### Dependency grammars



- No constituents, units of syntactic structure are words
- The structure of the sentence is represented by asymmetric binary relations between syntactic units
- The links (relations) have labels (dependency types)
- Each relation defines one of the words as the **head** and the other as **dependent**
- Often an artificial *root* node is used for computational convenience

### Dependency grammars: notational variation



## Dependency grammar: definition

A dependency grammar is a tuple  $(V, A)$

$V$  is a set of nodes corresponding to the (syntactic) words (we implicitly assume that words have indexes)

$A$  is a set of arcs of the form  $(w_i, r, w_j)$  where

$w_i \in V$  is the head

$r$  is the type of the relation (arc label)

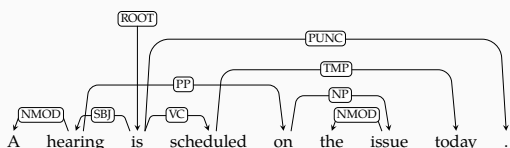
$w_j \in V$  is the dependent

This defines a directed graph.

## Dependency grammars: common assumptions

- Every word has a single head
- The dependency graphs are acyclic
- The graph is connected
- With these assumptions, the representation is a tree
- Note that these assumptions are not universal but common for dependency parsing

## Dependency grammars: projectivity



- If a dependency graph has no crossing edges, it is said to be *projective*, otherwise *non-projective*
- Non-projectivity stems from long-distance dependencies and free word order
- Projective dependency trees can be represented with context-free grammars
- In general, projective dependencies are parseable more efficiently

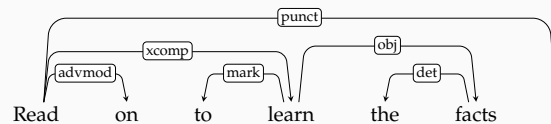
## CONLL-X/U format for dependency annotation

Single-head assumption allows flat representation of dependency trees

```

1 Read read VERB VB Mood=Imp|VerbForm=Fin 0 root
2 on on ADV RB - 1 advmod
3 to to PART TO - 4 mark
4 learn learn VERB VB VerbForm=Inf 1 xcomp
5 the the DET DT Definite=Def 6 det
6 facts fact NOUN NNS Number=Plur 4 obj
7 . . PUNCT . - 1 punct

```



example from English Universal Dependencies treebank

## Dependency parsing

- Dependency parsing has many similarities with context-free parsing (e.g., trees)
- They also have some different properties (e.g., number of edges and depth of trees are limited)
- Dependency parsing can be
  - grammar-driven (hand crafted rules or constraints)
  - data-driven (rules/model is learned from a treebank)
- There are two main approaches:
  - Graph-based similar to context-free parsing, search for the best tree structure
  - Transition-based similar to shift-reduce parsing (used for programming language parsing), but using greedy search for the best transition sequence

## Grammar-driven dependency parsing

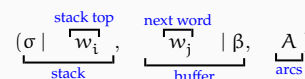
- Grammar-driven dependency parsers typically based on
  - lexicalized CF parsing
  - constraint satisfaction problem
    - start from fully connected graph, eliminate trees that do not satisfy the constraints
    - exact solution is intractable, often employ heuristics, approximate methods
    - sometime 'soft', or weighted, constraints are used
  - Practical implementations exist
- Our focus will be on data-driven methods

## Transition based parsing

- Inspired by shift-reduce parsing, single pass over the input
- Use a stack and a buffer of unprocessed words
- Parsing as predicting a sequence of transitions like
  - LEFT-ARC: mark current word as the head of the word on top of the stack
  - RIGHT-ARC: mark current word as a dependent of the word on top of the stack
  - SHIFT: push the current word to the stack
- Algorithm terminates when all words in the input are processed
- The transitions are not naturally deterministic, best transition is predicted using a machine learning method

(Yamada and Matsumoto 2003; Nivre, Hall, and Nilsson 2004)

## A typical transition system



LEFT-ARC<sub>r</sub>:  $(\sigma | w_i, w_j | \beta, A) \Rightarrow (\sigma, w_j | \beta, A \cup \{(w_i, r, w_i)\})$

- pop  $w_i$ ,
- add arc  $(w_i, r, w_i)$  to  $A$  (keep  $w_j$  in the buffer)

RIGHT-ARC<sub>r</sub>:  $(\sigma | w_i, w_j | \beta, A) \Rightarrow (\sigma, w_i | \beta, A \cup \{(w_i, r, w_j)\})$

- pop  $w_i$ ,
- add arc  $(w_i, r, w_j)$  to  $A$ ,
- move  $w_i$  to the buffer

SHIFT:  $(\sigma, w_j | \beta, A) \Rightarrow (\sigma | w_j, \beta, A)$

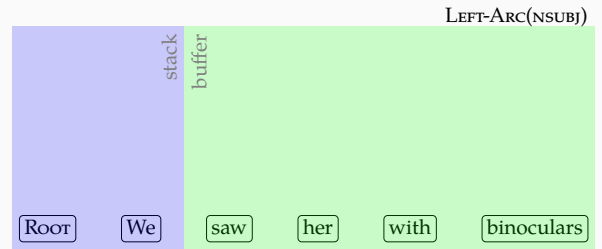
- push  $w_j$  to the stack
- remove it from the buffer

(Kübler, McDonald, and Nivre 2009, p.23)

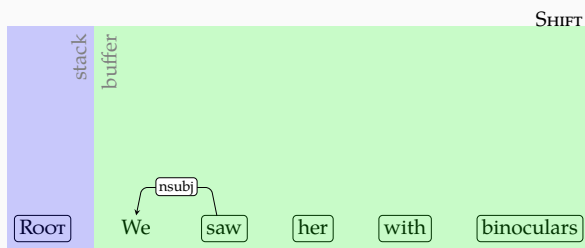
## Transition based parsing: example



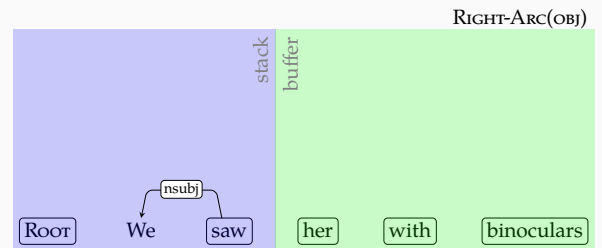
## Transition based parsing: example



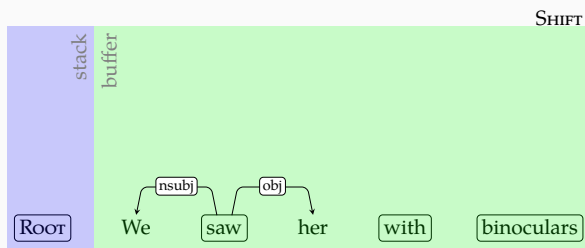
## Transition based parsing: example



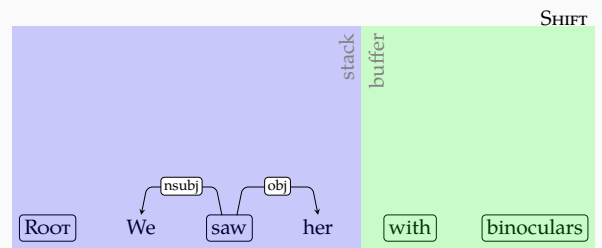
## Transition based parsing: example



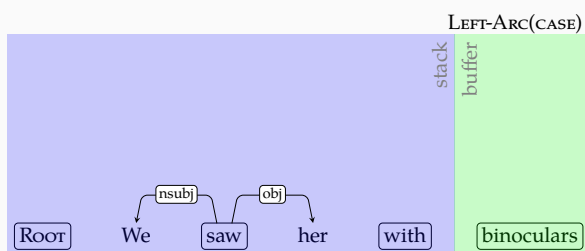
## Transition based parsing: example



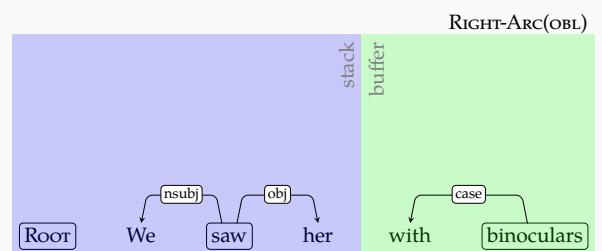
## Transition based parsing: example



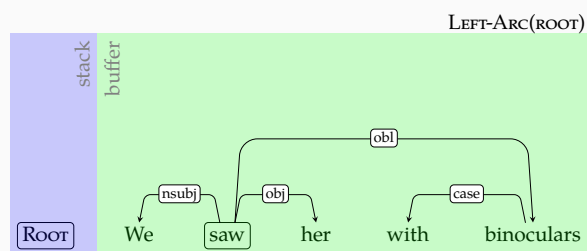
## Transition based parsing: example



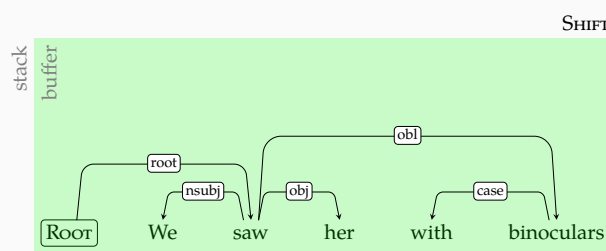
## Transition based parsing: example



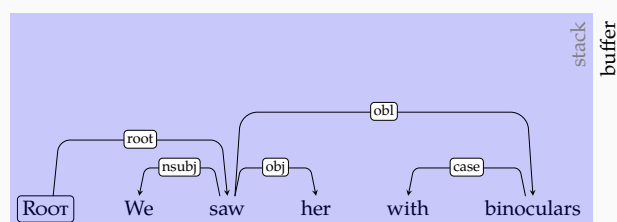
## Transition based parsing: example



## Transition based parsing: example



## Transition based parsing: example



## Making transition decisions

- In classical shift-reduce parsing the actions are deterministic
- In transition-based dependency parsing, we need to choose among all possible transitions
- The typical method is to train a (discriminative) classifier trained on features extracted from gold-standard *transition sequences*
- Almost any machine learning method method is applicable. Common choices include
  - Memory-based learning
  - Support vector machines
  - (Deep) neural networks

## Features for transition-based parsing

- The features come from the parser configuration, for example
  - The word at the top of the stack, (peeking towards the bottom of the stack is also fine)
  - The first/second word on the buffer
  - Right/left dependents of the word on top of the stack/buffer
- For each possible 'address', we can make use of features like
  - Word form, lemma, POS tag, morphological features, word embedding
  - Dependency relations –  $(w_i, r, w_j)$  triples
- Note that for some 'address'-'feature' combinations and in some configurations the values may be missing

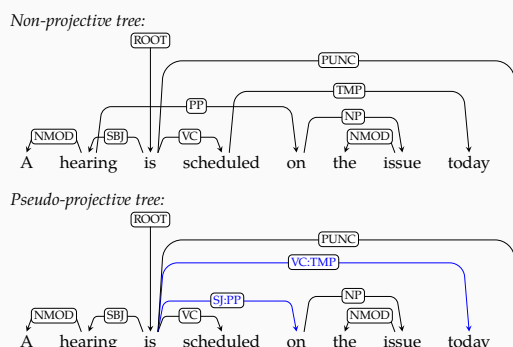
## The training data

- The features for transition-based parsing have to be extracted from *parser configurations*
- The data (treebanks) need to be preprocessed for obtaining the training data
- Construct a transition sequence by parsing the sentences, and using treebank annotations (the set  $A$ ) as an 'oracle'
- Decide for
  - LEFT-ARC<sub>T</sub> if  $(\beta[0], r, \sigma[0]) \in A$
  - RIGHT-ARC<sub>T</sub> if  $(\sigma[0], r, \beta[0]) \in A$
  - and all dependents of  $\beta[0]$  are attached
  - SHIFT otherwise
- There may be multiple sequences that yield the same dependency tree, the above defines a 'canonical' transition sequence

## Non-projective parsing

- The transition-based parsing we defined so far works only for projective dependencies
- One way to achieve (limited) non-projective parsing is to add special LEFT-ARC and RIGHT-ARC transitions to/from non-top words from the stack
- Another method is pseudo-projective parsing:
  - preprocessing to 'projectivize' the trees before training
    - The idea is to attach the dependents to a higher level head that preserves projectivity, while marking it on the change on the new dependency
  - postprocessing for restoring the projectivity after parsing
    - Re-introduce projectivity for the marked dependencies

## Pseudo-projective parsing





## Transition based parsing: summary/notes

- Linear time, greedy parsing
- Can be extended to non-projective dependencies
- One can use arbitrary features,
- We need some extra work for generating gold-standard transition sequences from treebanks
- Early errors propagate, transition-based parsers make more mistakes on long-distance dependencies
- The greedy algorithm can be extended to beam search for better accuracy (still linear time complexity)

## Graph-based parsing: preliminaries

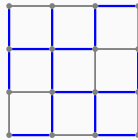
- Enumerate all possible dependency trees
- Pick the best scoring tree
- Features are based on limited parse history (like CFG parsing)
- Two well-known flavors:
  - Maximum (weight) spanning tree (MST)
  - Chart-parsing based methods

Eisner 1996; McDonald et al. 2005

## MST parsing: preliminaries

Spanning tree of a graph

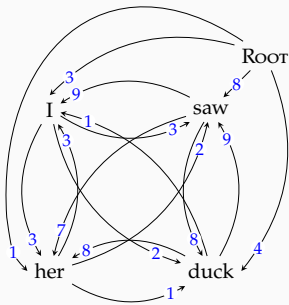
- Spanning tree of a connected graph is a sub-graph which is a tree and traverses all the nodes
- For fully-connected graphs, the number of spanning trees are exponential in the size of the graph
- The problem is well studied
- There are efficient algorithms for enumerating, and finding the optimum spanning tree on weighted graphs



## MST algorithm for dependency parsing

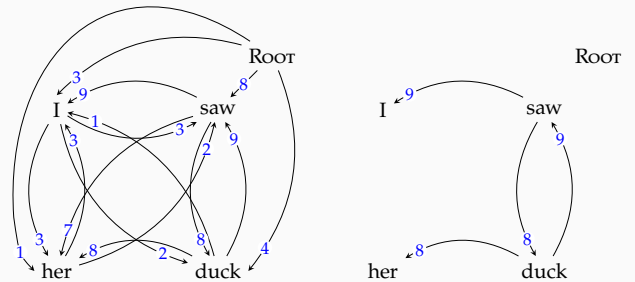
- For directed graphs, there is a polynomial time algorithm that finds the minimum/maximum spanning tree (MST) of a fully connected graph (Chu-Liu-Edmonds algorithm)
- The algorithm starts with a dense/fully connected graph
- Removes edges until the resulting graph is a tree

## MST example



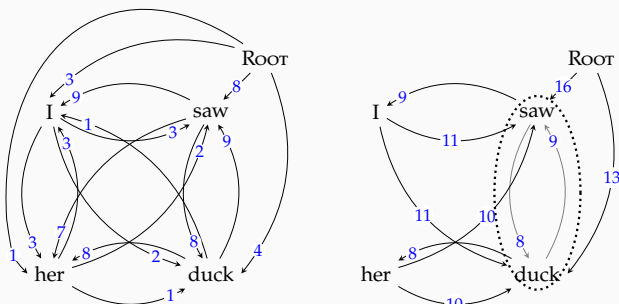
For each node select the incoming arc with highest weight

## MST example



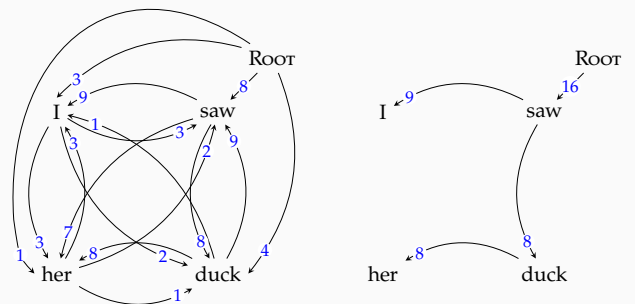
Detect the cycles, contract them to a 'single node'

## MST example



Pick the best arc into the combined node, break the cycle

## MST example



Once all cycles are eliminated, the result is the MST

## Properties of the MST parser

- The MST parser is non-projective
- There is an algorithm with  $O(n^2)$  time complexity (Tarjan 1977)
- The time complexity increases with typed dependencies (but still close to quadratic)
- The weights/parameters are associated with edges (often called 'arc-factored')
- We can learn the arc weights directly from a treebank
- However, it is difficult to incorporate non-local features

## CKY for dependency parsing

- The CKY algorithm can be adopted to projective dependency parsing
- For a naive implementation the complexity increases drastically  $O(n^6)$ 
  - Any of the words within the span can be the head
  - Inner loop has to consider all possible splits
- For projective parsing, the observation that the left and right dependents of a head are independently generated reduces the complexity to  $O(n^3)$

(Eisner 1997)

## Non-local features

- The graph-based dependency parsers use edge-based features
- This limits the use of more global features
- Some extensions for using 'more' global features are possible
- This often leads non-projective parsing to become intractable

## External features

- For both type of parsers, one can obtain features that are based on unsupervised methods such as
  - clustering
  - dense vector representations
  - alignment/transfer from bilingual corpora/treebanks

(Koo, Carreras, and Collins 2008)

## Errors from different parsers

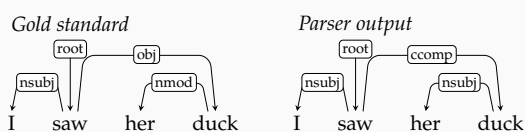
- Different parsers make different errors
  - Transition based parser do well on local arcs, worse on long-distance arcs
  - Graph based parser tend to do better on long-distance dependencies
- Parser combination is a good way to combine the powers of different models. Two common methods
  - Majority voting: train parsers separately, use the weighted combination of their results
  - Stacking: use the output of a parser as features for another

(McDonald and Satta 2007; Sagae and Lavie 2006; Nivre and McDonald 2008)

## Evaluation metrics for dependency parsers

- Like CF parsing, exact match is often too strict
- *Attachment score* is the ratio of words whose heads are identified correctly.
  - *Labeled attachment score* (LAS) requires the dependency type to match
  - *Unlabeled attachment score* (UAS) disregards the dependency type
- *Precision/recall/F-measure* often used for quantifying success on identifying a particular dependency type
  - precision is the ratio of correctly identified dependencies (of a certain type)
  - recall is the ratio of dependencies in the gold standard that parser predicted correctly
  - f-measure is the harmonic mean of precision and recall
$$\left( \frac{2 \times \text{precision} \times \text{recall}}{\text{precision} + \text{recall}} \right)$$

## Evaluation example



UAS	100%
LAS	50%
Precision <sub>nsubj</sub>	50%
Recall <sub>nsubj</sub>	100%
Precision <sub>obj</sub>	0% (assumed)
Recall <sub>obj</sub>	0%

## Averaging evaluation scores

- As in context-free parsing, average scores can be
  - macro-average or sentence-based
  - micro-average or word-based
- Consider a two-sentence test set with
 

	words	correct
sentence 1	30	10
sentence 2	10	10

  - word-based average attachment score: 50% (20/40)
  - sentence-based average attachment score: 66% ((1 + 1/3)/2)

## Dependency parsing: summary

- Dependency relations are often semantically easier to interpret
- It is also claimed that dependency parsers are more suitable for parsing free-word-order languages
- Dependency relations are between words, no phrases or other abstract nodes are postulated
- Two general methods:
  - transition based greedy search, non-local features, fast, less accurate
  - graph based exact search, local features, slower, accurate (within model limitations)
- Combination of different methods often result in better performance
- Non-projective parsing is more difficult
- Most of the recent parsing research has focused on better machine learning methods (mainly using neural networks)

## Next

Wed Unsupervised learning

Fri Exercises (+ second graded assignment)

## Where to go from here?

- Müller (2016) is a new open-source text book on Grammar formalisms.
- Aho and Ullman (1972) is the classical reference (available online) for parsing (programming languages) and also includes discussion of grammar classes in the Chomsky hierarchy. A more up-to-date alternative is Aho, Lam, et al. (2007).
- There is a brief introductory section on dependency grammars in Kübler, McDonald, and Nivre (2009), for a classical reference see Tesnière (2015), English translation of the original version (Tesnière 1959).

## Pointers to some treebanks

Treebanks are the main resource for statistical parsing. A few treebank-related resources to have a look at until next time:

- Tübingen treebanks:
  - TüBa-D/Z written German
  - TüBa-D/S spoken German
  - TüBa-E/S spoken English
  - TüBa-J/S spoken Japanese
 available from <http://www.sfs.uni-tuebingen.de/en/ascl/resources/corpora.html>
- Universal dependencies project, documentation, treebanks: <http://universaldependencies.org/>
- TüNDRA - a treebank search and visualization application with the above treebanks and few more
  - Main version: <https://weblicht.sfs.uni-tuebingen.de/Tundra/>
  - New version (beta): <https://weblicht.sfs.uni-tuebingen.de/tundra-beta/>

## CKY algorithm

```

function CKY(words, grammar)
  for j ← 1 to LENGTH(words) do
    table[j - 1, j] ← {A | A → words[j] ∈ grammar}
  for i ← j - 1 downto 0 do
    for k ← i + 1 to j - 1 do
      table[i, j] ← table[i, j] ∪
        {A | A → BC ∈ grammar and
          B ∈ table[i, k] and
          C ∈ table[k, j]}
  return table

```